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F. Cerulus, Remarks on the classification of fundamental particles 557
A. N. Mitra, Multiple π^0-production in anti-proton annihilation 571
A. J. F. Boyle, The spin of the 2.9 MeV state of Be^8 . . 581
L. I. Dorman, N. S. Kaminer, V. K. Koiava, Yu. G. Shafer and B. F. Schwarzman, Observation of the large cosmic ray increase of February 23, 1956 in the U.S.S.R.. 585
A. Bisi, E. Germagnoli and L. Zappa, L/K-Capture ratio in ^{153}Gd and low-lying levels in ^{153}Eu 593
M. Roclawski-Conjeaud et E. Cotton, Courbes d'excitation de la réaction $^{16}O(d,p)^{17}O^*$ 603
A. S. Meligy, The wave functions in coulomb fields . . . 610
S. A. Wouthuysen, Impressions of the 6th Rochester Conference 619
L. van Hove, George Placzek 623
News items 627

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REMARKS ON THE CLASSIFICATION OF FUNDAMENTAL PARTICLES

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Abstract: Starting from a few experimentally observed reactions and the assumed existence of strong, electromagnetic and weak interactions, a correspondence between all fundamental particles and a certain Abelian group is set up, without referring to any details of the interaction. This formalism allows the derivation of the selection rules for the production of hyperons, K-mesons and π-mesons. The weak interactions of K-mesons, pions, muons, and leptons are correctly described by the formalism, and the conservation of light fermions is derived from it. The relation between the rules for the production and those for the decay of the hyperons and K-mesons is explained.

Of late several sets of rules have been proposed to explain in a formal way the interactions and decays of hyperons and K-mesons. [1,2,3,4])

D'Espagnat and Prentki [5]) have shown that these are equivalent † and may be derived from the conservation of baryons and the charge-independence of the Lagrangian.

It is the purpose of this article to point out that those rules may be derived without any reference to the form or the symmetries of the interaction, using only a few experimentally observed reactions and a general principle relating the experimental facts and the mathematical formalism.

It will appear afterwards that this principle may be used to get selection rules for the much weaker interactions between muons and leptons; the weak interactions of pions and K-mesons can be fitted into the scheme, and at this point an explanation of the decays of the hyperons and their relation to the production will suggest itself. It will then appear that all fundamental particles,

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† The rules of Salam and Polkinghorne [4]) differ from the others in treating the τ-meson as distinct from the θ-meson. Their quantum number μ_3 is $\frac{1}{2}U$, in the notation of ref. [5]), for hyperons and θ-mesons.

including the photon, can be brought into relation with a simple group, which summarises the possible interactions between them.

An extensive use of the terms "strong interactions" and "weak interactions" will be made in this article, and they will henceforth be abbreviated by S.I. and W.I. For a discussion on the experimental evidence of the existence of these two well-separated classes of interaction, and a further qualification of those terms, the reader is referred to the discussions of d'Espagnat and Prentki [5]) and Nishijima [6]).

The W.I. will give rise to slow reactions, mainly decays with a half-life of 10^{-10} s or longer; a reaction will be called fast, on the other hand, if it takes place in a time comparable to $\hbar/Mc^2 \approx 10^{-21}$ s (M denotes the nucleon or hyperon mass).

The proposed scheme embodies the law of the conservation of charge, and a few other conservation laws which have been enuntiated more recently. But there are of course still other conservation laws, those of energy or of total parity, e.g., which are foreign to the present formalism and may forbid transitions allowed by it.

If a reaction is not forbidden by any selection rule, and if it can occur through a sequence of S.I. then it will be observed to be a fast reaction.

A similar statement about the W.I. cannot be made: every W.I. in a sequence of interactions leading to a given reaction, lowers the total reaction rate by many orders of magnitude, and the occurrence of two W.I. in a reaction could very well render it almost unobservable. So, unless there exists a means of deciding how many W.I.-steps are involved in a reaction, the selection rules are not able to predict which of the allowed slow reactions will actually occur with a measurable rate.

1. The Strong Interactions

1.1. CLASSIFICATION OF THE PARTICLES

We know from experiment that the following reactions are fast, and should hence be ascribed to S.I.:

$$\mathrm{N}+\mathrm{N} \rightarrow \mathrm{N}+\mathrm{N}+\pi \tag{1.1}$$

$$\pi+\mathrm{N} \rightarrow \Sigma+\mathrm{K}^{+,0} \tag{1.2}$$

$$\pi+\mathrm{N} \rightarrow \Lambda+\mathrm{K}^{+,0} \tag{1.3}$$

$$\mathrm{K}^{-}+\mathrm{N} \rightarrow \Sigma+\pi \tag{1.4}$$

$$\mathrm{K}^{-}+\mathrm{N} \rightarrow \Lambda+\pi; \tag{1.5}$$

(1.2) and (1.3) are the so-called associated production processes, many examples of which are known [7]); (1.4), (1.5) follow from the experiments of Fry *et al.* [8]).

The great positive excess in K-production and the large absorption cross-section for K^-, compared to that for K^+, lead us to consider the $K^{+,0}$ on the one hand, and the K^- on the other, as having different S.I.. We shall usually call the couple $K^{+,0}$ K_1-mesons, while the K^- will be called K_2.

There is some evidence for the associated production of Ξ^- with two K^0-mesons [9]). We therefore tentatively assume the occurrence of the fast reaction

$$\pi^- + N^0 \rightarrow \Xi^- + K^0 + K^0. \qquad (1.6).$$

From these facts it is possible to derive a mathematical formalism if we assume the following postulates:

I. To every particle having strong interactions a (mathematical) element is associated.

II. These elements combine in such a way that the "product" of elements corresponding to the particles resulting from a fast reaction is equal to the "product" of the elements of the initial particles.

The commutative law should of course be valid for this product, as the order of the emerging particles is arbitrary.

Postulate I implies that different elements are related to different particles, but the converse need not always hold.

The elements corresponding to the particles that occur in (1.1) to (1.6) form a certain Abelian group G.

To π corresponds the unit E of G, because of (1.1).

The element corresponding to K_2 is called A, and the one corresponding to N is called B, by convention.

Because of the pair of reactions (1.3) and (1.5) (or (1.2) and (1.4)) A^{-1} corresponds to K_1.

The annihilation of a nucleon pair with production of π's compels us to assign B^{-1} to the antinucleon.

Because of (1.2) and (1.3) BA corresponds to both Λ and Σ.

If (1.6) is taken for granted BA^2 should be assigned to the Ξ.

The anti-hyperons, although not yet observed, should find a place in the scheme as we believe them to have strong interactions too. If the K_2 is assumed to be the charge-conjugate particle of the K_1 (this involves the existence of a $\overline{K^0}$ different from K^0), charge conjugation applied to both sides of eq. (1.2), e.g., gives

$$\pi + \overline{N} \rightarrow \overline{\Sigma} + K_2 \qquad (1.7)$$

and to the anti-Σ must correspond the element $B^{-1}A^{-1}$. Similarly, to $\overline{\Lambda}$ corresponds $B^{-1}A^{-1}$ and to $\overline{\Xi}$, $B^{-1}A^{-2}$. Table 1 summarizes these correspondences.

TABLE 1

Element	Particle	Element	Particle
E	π	BA	Λ, Σ
A	K_2 (= $K^{-,0}$)	$B^{-1}A^{-1}$	$\overline{\Lambda}$, $\overline{\Sigma}$
A^{-1}	K_1 (= $K^{0,+}$)	BA^2	Ξ
B	N	$B^{-1}A^{-2}$	$\overline{\Xi}$
B^{-1}	$\overline{N}$		

The element of an antifermion, in this table, is in every case the inverse of the fermion element.

1.2. INTERPRETATION AND CONSEQUENCES OF THE CLASSIFICATION

Table 1, combined with postulate II, leads to the known rules of Gell-Mann and Nishijima. To every state of particles is associated an element of the general form $A^\alpha B^\beta$, and all states which can be reached through S.I. from this initial state must have the same element. This means that α and β (which may be integers of either sign or zero) are quantum numbers for the S.I.; they are associated with two already proposed conservation laws.

All baryon elements contain a factor B and all anti-baryons a factor B^{-1}; the exponent β denotes, therefore, the number of baryons minus the number of anti-baryons.

The quantum number α is equal to the attribute of Sachs [3]), and the combination $\beta - \alpha$ is equal to the U of d'Espagnat and Prentki [5]). The law of conservation of iso-fermions proposed by the last authors can be deduced from table 1. Let x, y, z, u, in this order, be the difference of the numbers or K_2 and K_1, N and $\overline{N}$, Λ or Σ and $\overline{\Lambda}$ or $\overline{\Sigma}$, Ξ and $\overline{\Xi}$ which are present in some initial state; these numbers are taken positive for an excess of K_2 over K_1 or N over $\overline{N}$, etc.

The element of this state is then

$$A^x \cdot B^y \cdot A^z B^z \cdot A^{2u} B^u$$

and α and β are, expressed in these numbers,

$$\alpha = x+z+2u$$
$$\beta = y+z+u;$$

the combination

$$\beta-\alpha = -x+y-u$$

is of course a quantum number for the interaction. It means simply that the number of K_1, N, and $\bar{\Xi}$, the so-called iso-fermions, minus that of their antiparticles, is conserved in S.I.; this rule was first derived by d'Espagnat and Prentki from the properties of their interaction-Lagrangian.

Table 1 predicts the existence of other reactions due to the S.I., as e.g.

$$\pi+N \to N+K_1+K_2 \tag{1.8}$$
$$\Lambda+K_2 \to \Xi\ (+\pi) \tag{1.9}$$
$$\Sigma+K_2 \to \Xi\ (+\pi) \tag{1.10}$$
$$\Xi \leftrightarrows \Xi+\pi \tag{1.11}$$
$$\Sigma \leftrightarrows \Sigma+\pi \tag{1.12}$$
$$\Sigma \leftrightarrows \Lambda+\pi \tag{1.13}$$
$$\Lambda \leftrightarrows \Lambda+\pi. \tag{1.14}$$

The associated production of two K-mesons (1.8) has recently been observed [10,11]). Reactions (1.11) to (1.14) predict "nuclear forces" between hyperons; the last one is decidedly allowed by the scheme, although it is forbidden by the conservation of total isobaric spin if one makes the usual assignment (cf. ref. [5])) $I = 0$ to the Λ. Its occurrence would possibly affect significantly the binding energy of the Λ-hyperfragments.

1.3. CHARGE CONSERVATION

In all the reactions considered so far we have assumed as a matter of course that only those which conserved charge, in addition to the G-element, could occur. By a trivial extension of G this fundamental law can be included in the formalism, which will then allow the fast reactions between given particles to be read off almost automatically.

Every element belonging to a positive particle will be multiplied by a new element, say C, every negative particle element by C^{-1}, the neutral particles have their elements unchanged.

The correspondence is displayed in table 2.

TABLE 2

Particles	Elements	Particles	Elements
π^+, π^0, π^-	C, E, C^{-1}	Λ^0 and Σ^0	AB
K^+, K^0	$A^{-1}C$, A^{-1}	$\overline{\Lambda^0}$ and $\overline{\Sigma^0}$	$A^{-1}B^{-1}$
$\overline{K^0}, K^-$	A, AC^{-1}	Σ^+, Σ^-	ABC, ABC^{-1}
N^+, N^0	BC, B	$\overline{\Sigma^-}, \overline{\Sigma^+}$	$A^{-1}B^{-1}C^{-1}$, $A^{-1}B^{-1}C$
$\overline{N^0}, \overline{N^-}$	B^{-1}, $B^{-1}C^{-1}$	Ξ^-	BA^2C^{-1}
		$\overline{\Xi^+}$	$B^{-1}A^{-2}C$

The group G is evidently homomorphic to this new group, which will be called G_e.

The exponent of C, say γ, is nothing but the total charge of the particles present in a given state with element $A^\alpha B^\beta C^\gamma$. For the correspondence between the quantum numbers $\beta-\alpha$, γ and the isobaric spin we refer to the discussion by d'Espagnat and Prentki[5]), §4. In the present notation

$$I_3 = \gamma + \frac{\alpha-\beta}{2}.$$

It may be pointed out that in the framework of the G_e-scheme there is no need to attach a special physical meaning to this particular combination of α, β, and γ.

1.4. FORMAL RULE FOR THE DECAYS

Sachs [3]) and Nishijima [6]) have in fact shown that the group G_e may be used to describe the decays of hyperons and some K-mesons if we picture the decay as the substitution of an E for an A in the element of the decaying particle.

This prescription corresponds to their selection rule

$$|\Delta a| = |\Delta \eta| = 1$$

for the W.I.. It may be seen to account correctly for the hyperon decays, especially the occurrence of

$$\Xi^- \to \Lambda^0 + \pi^-$$

and the absence of

$$\Xi^- \to N^0 + \pi^-.$$

This fact is in favour of the assignment of A^2BC^{-1} to Ξ^-.

It will be seen in the next section that this ad hoc rule can be explained by considering the whole of the weak interactions. In the

framework of this scheme it is still possible for the τ and the χ-mesons to be different particles, provided they have the same strong interactions.

2. The Weak Interactions

2.1. MUONS AND LEPTONS

The usefulness of the present formalism in classifying the experimental evidence is not limited to the particles having S.I., but the weakly interacting particles fit into an analogous scheme, based on the same group G_e.

The correspondence-table can be derived, for muons and leptons, from the following well-known reactions:

$$N^+ + \mu^- \rightarrow N^0 + (\nu) \tag{2.1}$$

$$N^0 \rightarrow N^+ + \beta^- + (\nu) \tag{2.2}$$

The brackets (ν) denote either a neutrino or an antineutrino, as it is experimentally not possible to decide which is which. The postulates are similar to I and II but the expressions "strong interaction" and "fast reaction" are changed into "weak interaction" and "slow reaction". These forms of the postulates will be referred to as 1 and 2.

It will of course not be possible to avoid that some particles, like the nucleons, appear in both the S.I. and W.I. schemes, and it will be convenient to denote them by related symbols in both schemes. The N^0-element will therefore be called b and the N^+-element bc; a lower case type for all the W.I. elements will be used.

The neutrino-element we call a, the antineutrino a^{-1}. This choice of course does not imply any connection between K-mesons and neutrino's, but is made with the purpose to show clearly the similarity between the S.I. and the W.I. schemes; this question is taken up in section 3.

If we denote the μ^- element by x and the β^- by y, reactions (2.1) (2.2) lead to one of the following pairs of equations:

$$x = c^{-1}a^{-1} \qquad y = c^{-1}a \tag{2.3}$$

$$x = c^{-1}a \qquad y = c^{-1}a^{-1} \tag{2.4}$$

$$x = y = ac^{-1} \tag{2.5}$$

$$x = y = a^{-1}c^{-1}; \tag{2.6}$$

(2.3) and (2.4) differ only by an interchanging of the names neutrino and antineutrino, and so do (2.5) and (2.6). If (2.5) or (2.6)

were the right solution, it would imply that, as the μ^- and the β^- have the same element, the reaction

$$N^+ + \mu^- \rightarrow N^+ + e^- \tag{2.7}$$

should compete with (2.1). In fact (2.7) is by several orders of magnitude less frequent than (2.1), if it occurs at all [12]); (2.5) and (2.6) should therefore be rejected.

This step may be justified by arguments of elegance and economy: if (2.5) or (2.6) were adopted the absence of (2.7), (2.12) and (2.13) should be explained in every single case by referring to the details of the interactions that could lead to such reactions.

If, however, it is taken for granted that the W.I. of those light fermions are of the Fermi-type with approximately the same coupling constant for all (the "universal Fermi-interaction"), then every reaction involving four particles should occur in a single step, with only one W.I.

The arguments given in the introduction show that the allowed reactions should then indeed be observable, so that from the non-occurrence of (2.7), (2.12), and (2.13) it may be concluded that they are forbidden. Taking (2.3) to be the solution, the following correspondence table results:

TABLE 3

Particle	Element	Particle	Element
ν	a	$\bar{\nu}$	a^{-1}
β^+	ca^{-1}	β^-	$c^{-1}a$
μ^+	ca	μ^-	$c^{-1}a^{-1}$

Reactions (2.1) and (2.2) should now read

$$N^+ + \mu^- \rightarrow N^0 + \bar{\nu} \tag{2.8}$$

$$N^0 \rightarrow N^+ + \beta^- + \bar{\nu}. \tag{2.9}$$

Without further assumptions the muon-decay is given by

$$\mu^+ \rightarrow \beta^+ + 2\nu \tag{2.10}$$

$$\mu^- \rightarrow \beta^- + 2\bar{\nu}, \tag{2.11}$$

two like neutrino's being emitted. Table 3 forbids not only (2.7), but the following decays as well:

$$\mu^\pm \rightarrow \beta^\pm + \beta^+ + \beta^- \tag{2.12}$$

$$\mu^\pm \rightarrow \beta^\pm + \gamma. \tag{2.13}$$

The last line calls for a discussion of the relation of the electromagnetic interactions to the present formalism; this will be given in section 4.

If the neutrino were a Majorana particle this scheme would break down, as then there is no distinction between a and a^{-1}.

The exponents of a, b, and c are quantum numbers of the W.I., and their physical meaning in terms of conservation laws is quite simple.

The exponent of c denotes the total charge, and the exponent of b the number of nucleons minus antinucleons, two quantities which are conserved by both S.I. and W.I.

The exponent of a is equal to the number of particles having a factor a in their element, minus the number of particles having a factor a^{-1}. This means that the number of ν, β^- and μ^+ minus the number of $\bar{\nu}$, β^+ and μ^- is conserved in the weak interactions. The last rule is nothing but the Konopinski-Mahmoud rule of conservation of light fermions [13]). All the physical results of the W.I.-scheme, e.g. (2.7) (2.10) (2.11) (2.12) agree with those of this theory, and we refer the reader to it for a discussion of the physical implications.

2.2. PIONS

The present formalism avoids referring to the interactions except in a qualitative way, and then only to distinguish, order-of-magnitude-wise, between S.I. and W.I.

From this point of view it is then tempting to include the pion decay into the scheme as some W.I. is certainly involved in it.

As the pion has S.I. too, we are restricted in assigning an element to it: because of these the pairs of states

$$\begin{aligned} &\pi^+ \text{ and } N^+ + N^0 \\ &\pi^- \text{ and } N^0 + \overline{N^-} \\ &\pi^0 \text{ and } N^+ + \overline{N^-} \\ &\pi^0 \text{ and } N^0 + \overline{N^0} \end{aligned}$$

are equivalent, and should therefore have the same element in the W.I.-scheme. Assigning to anti-particles inverse elements, this means that

$$\begin{aligned} &\pi^+ \text{ has the element } c, \\ &\pi^- \quad ,, \quad ,, \quad ,, \quad c^{-1}, \\ &\pi^0 \quad ,, \quad ,, \quad ,, \quad e. \end{aligned}$$

Many decay reactions of the π^0 suggested by the last line will be unobservable, due to the rapid γ-decay of the π^0. Its inclusion into the W.I.-scheme is, however, essential to the explanation of the K-meson decays.

The present assignment allows the normal pion decay

$$\pi^+ \to \mu^+ + \bar{\nu} \tag{2.14}$$

$$\pi^- \to \mu^- + \nu. \tag{2.15}$$

The pion β-decay is on the other hand not forbidden, and should be written

$$\pi^+ \to \beta^+ + \nu \tag{2.16}$$

$$\pi^- \to \beta^- + \bar{\nu}. \tag{2.17}$$

A satisfactory explanation for the rarity (or non-occurrence) of the $\pi-\beta$ decay will probably only be found by considering in detail the interactions responsible for it. The formulae (2.14), (2.15) and (2.16), (2.17) point out that there should be an essential difference between the two modes of decay, one yielding a ν, the other a $\bar{\nu}$.

2.3. K-MESONS

Another set of phenomena where the W.I. should play a role, is given by the decays of the K-mesons. It will be seen that they too fit into the scheme. We list in the next table their known decay modes and, in the right-hand column, the elements to which these correspond.

TABLE 4

K-mesons	Elements
$K^+_{\mu 2} \to \mu^+ + (\nu)$	c or ca^2
$K^+_{\pi 2} \to \pi^+ + \pi^0$	c
$\tau^+ \to \pi^+ + \pi^+ + \pi^-$	c
$\tau'^+ \to \pi^+ + \pi^0 + \pi^0$	c
$K^+_{\mu 3} \to \mu^+ + (\nu) + \pi^0$	c or ca^2
$K^+_{\beta} \to e^+ + ? + ?$	?
$\theta^0 \to \pi^+ + \pi^-$	e
$\tau^- \to \pi^- + \pi^+ + \pi^-$	c^{-1}

It seems reasonable to assume that all K^+-mesons have the element c, which is in agreement with all available evidence.

This means that

$$K^+_{\mu 2} \to \mu^+ + \bar{\nu}$$

$$K^+_{\mu 3} \to \mu^+ + \bar{\nu} + \pi^0.$$

According to table 4 the K_β^+ may decay into $e^+ + \nu$ or $e^+ + \nu + \pi^0$ or $e^+ + \nu + 2\pi^0$; the $K_{\mu 3}^+$ decay into $\mu^+ + \bar{\nu} + 2\pi^0$ is not excluded by it.

2.4. THE DECAY OF THE HYPERONS

The scheme could apparently be built up still further by bringing the hyperons into it, using their decays to find out the correspondence between hyperons and elements. This does not, however, meet with success, as can be seen from the following list:

$$\begin{array}{ll} \Lambda^0 \to N^+ + \pi^- & b \\ \Sigma^0 \to \Lambda^0 + \gamma & b \\ \Sigma^+ \to \begin{cases} N^+ + \pi^0 \\ N^0 + \pi^+ \end{cases} & bc \\ \Sigma^- \to N^0 + \pi^- & bc^{-1} \\ \Xi^- \to \Lambda^0 + \pi^- & bc^{-1}. \end{array}$$

The last line suggests that the Ξ^- might decay as well into $N^0 + \pi^-$. This difficulty can however be avoided, if it is assumed that the hyperons have no weak interactions, except through the K-mesons. In this case they have no element in the W.I.-scheme, just as the leptons have none in the S.I.-scheme. This assumption accounts fully for the observed decays because in the W.I.-scheme the K-mesons have the same element as the π-mesons, which they have not in the S.I.-scheme.

This means that the W.I. will violate the S.I. relation rules, and their effect is to make a state corresponding to A go over into one corresponding to E; it was shown in subsection 1.3 that this is exactly how the hyperon-decays are formally described. It is not claimed that this is the only possible explanation for the rule in 1.3. For the Ξ-decay it is only required that the Ξ, N and π are practically not directly coupled. Couplings between Σ, Λ, N, and π may be imagined which would yield the same rule; see e.g. ref. [15]) or [16]) for such proposals.

The fact that the A^2 in the Ξ-element does not go over into E, but into AE can be explained if we make the obvious assumption that this can only happen through two subsequent W.I., so that this decay-rate should be several orders of magnitude smaller than the observed one.

Still, the modes of decay of the hyperons suggested by this formalism are more numerous than the observed ones; e.g.

$$\Sigma^+ \to N^0 + \mu^+ + \bar{\nu}$$

would be an allowed transition. Experimentally it would be interesting to know whether this decay happens, and also whether the decay

$$\Lambda^0 \rightarrow N^0+\pi^0$$

occurs with a measurable frequency, as this last one is the only 2-body hyperon decay that is allowed † and has not been observed hitherto.

Some calculations of the relative frequencies of these unobserved hyperon decays and some K-decays have been made by Dallaporta[14]) and Duimio [15]), who considered the W.I. to be a universal Fermi-interaction. The last author reviews the experimental evidence for the different W.I. hitherto proposed.

3. Unified Formulation of the Scheme

Comparing the S.I. and the W.I. schemes, it is seen that they can in fact be derived from a single group Γ, built up of the products of the fundamental elements E, B, C, A, a, and their inverses. The reason for this is that particles appearing in both schemes have corresponding elements, and as the elements B and b or C and c give rise to the same conservation law, they can be identified. Only the K-mesons have two elements for each particle, one to be used with postulates I, II for the S.I., the other with postulates 1, 2 for the W.I..

The elements containing A (or A^{-1}) are to be used only for conclusions about the S.I., those containing a or a^{-1} only for W.I., those containing neither can be used for both.

4. The Electromagnetic Interactions

It is possible, in principle, to construct a correspondence table for the electromagnetic interactions along the same lines as those for the S.I. and W.I.. But no new quantum number will result from it, for the following reason: if a particle P emits γ-rays,

$$P \rightarrow P'+\gamma, \tag{4.1}$$

P′ is usually called the same kind of particle as P, and therefore γ corresponds to the unity element of the group.

Reactions as (4.1) do not lead to relations between different kinds of particles, and it is not possible in general to express the element

† The fast decay of Σ^0 into $\Lambda^0+\gamma$ would mask any other mode, and the other decays of hyperons, suggested by the scheme, are energetically forbidden.

of one kind of particle as a product of the elements of different kinds.

In order to draw up a correspondence table it is then necessary to assign to almost every kind of positive particle a different fundamental element, playing a role similar to that of B or C in the G_e-scheme. The inverses of those elements then correspond to the negative particles. Those particles which, although of a different kind, could decay into each other with photon emission would then have the same element (Σ^0 and Λ^0, e.g.). The neutral particles, with the exception of π^0, have no place in such a scheme, and the reactions caused by their magnetic moments do not follow from it.

The π^0, because of its γ-decay, has to have the same element as the γ, i.e. unity.

The arrangement just sketched accounts for all decays like (4.1) and for pair creation. The pair creation, however, considered apart from the other e.m. interactions could be described by means of the Γ-scheme if E is assigned to the photon. This is possible because particles of opposite sign have inverse elements in this scheme, so that Γ is homomorphic to the group that would describe fully the electric interactions, and in this last group the γ and the π^0 correspond already to the same element.

With the electric interactions described by the Γ-scheme, the decay (2.13) is indeed forbidden. All reactions (4.1) are allowed in this arrangement too; however their reaction rates will have another order of magnitude, intermediate between the slow and the fast ones; this fact should be borne in mind when using the scheme for reactions involving e.m. interactions.

5. Conclusions

As the previous investigation is entirely based on the two postulates I and II in 1.1, it is perhaps worth while to say a few words about their significance. As it has been seen, the essential assumption made there is that the quantum numbers governing the reactions from a definite state can be obtained through a simple algebraic addition of the quantum numbers for the particles forming the state in question. This might seem to be a very general statement but e.g. the very important vector addition rules for angular momenta cannot be described in this way. As much recent work in the field of elementary particles has been based on the concept of isobaric spin which is itself modelled on that of angular momentum, it is not self-

evident that the much simpler approach investigated here will lead to a satisfactory description of the experimental data.

Nevertheless, it turns out that this is indeed so, a fact that is of course intimately connected with the simple law for the conservation of the third component of the usual isobaric spin vector. Only in very few cases does the restriction on the magnitude of the isobaric spin involved lead to significant deviations from the rules obtained here. The only instance where this could possibly be of practical importance is the reaction (1.14) but so far there seems to be no evidence for or against it.

Perhaps it could be mentioned further that the only fact indicating that this naive approach is not sufficient is the forbiddenness of the β-decay of the π-meson. This may indicate the need for a more sophisticated approach to the selection rules, unless the large branching ratio between (2.14) and (2.16) can be explained in some less fundamental way, e.g. by assuming pseudo vector interaction [17,18]). Which solution is the right one remains to be seen.

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MULTIPLE π^0-PRODUCTION IN ANTI-PROTON ANNIHILATION

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Abstract: The Feynman-Dyson formalism is applied to the calculation of multiple π^0-production in proton-antiproton annihilation. The calculations are along lines very similar to those recently followed by Gupta. It is found that contrary to the corresponding results of Quantum Electrodynamics, the cross section for the multiple production of π^0's begins to decrease only for values of $n > 15$. Since each π^0 decays into two photons almost immediately, the bearing of this result on the Schein event is briefly discussed.

1. Introduction

The observation by Schein [1]) and co-workers of a narrow shower of a large number of photons, and subsequently by others [2]) of similar events, has aroused considerable interest in theoretical circles. Gupta [3]) recently investigated the event from the point of view of multiple photon production in quantum electrodynamics. He found that though for extremely large energies ($\approx 10^{14}$ eV), the effect of the small factor $(e^2/4\pi\hbar c)$ was largely compensated by large logarithmic factors in energy, the cross section for the multiple production of photons was still small because of the appearance of an additional factor $(n-2)!$ in its denominator.

A possible interpretation which was given by Schein in his paper [1]) was that the event might be due to some annihilation process involving a heavy particle moving with a very high velocity. Now the simplest anti-particle (next to the positon) that one might consider is the anti-proton, interest in which has been revived in recent times on account of its actual production in the laboratory. Thus it might be useful to consider in some detail the annihilation of a proton by an anti-proton (moving with a very large velocity), giving rise to a large number of mesons. The idea of multiple meson production in nucleon-nucleon collision is by no means a new one, and the problem is known to be complicated on account of the largeness of the meson coupling constant. It is felt, however, that

there is some difference between the process of meson production in nucleon-nucleon collision and that in nucleon-antinucleon annihilation, the difference being roughly the one that exists between one-body and two-body problems. This can be most easily realized with the help of a "Feynman diagram" in which an "anti-particle" is represented by the corresponding "particle" travelling *backward* in time. Thus though the problem of meson production in nucleon-nucleon collision has been studied in the past, it is not obvious that all the considerations applying to that process will be applicable without modification to the case of anti-nucleon annihilation as well. In any case it seems desirable to calculate the cross section for multiple production of mesons in anti-nucleon annihilation, even by straight-forward perturbation theory.

For simplicity we shall in this paper consider only the production of neutral pions, using the covariant Feynman-Dyson formalism for pseudoscalar theory. In this connection it may be noted that the production of neutral pions has a special significance with respect to the Schein event, since a π^0 decays almost immediately into two photons. The calculations will be on lines closely following those of Gupta [3]) whose paper will be referred to as A. We shall therefore omit as much as possible the details of various considerations employed by Gupta, except for certain modifications which are bound to occur in the present case, on account of the different nature of the interaction involved.

2. Two π^0 Annihilation

The Hamiltonian density can be taken as

$$H(x) = iG\bar{\psi}(x)\gamma_5\psi(x)\phi(x), \tag{1}$$

where we have put $\tau_3 = 1$ and $\phi(x)$ denotes the wave function of a π^0. In what follows, we shall put $\hbar = c = 1$.

Using Dyson's procedure [4]), one obtains the S-matrix element for the process of proton-antiproton annihilation with the emission of two π^0's in the form

$$S_2 = G^2 \int dx dx' \phi(x)\phi(x')\bar{\psi}(x')\gamma_5 S_F(x-x')\gamma_5\psi(x), \tag{2}$$

where

$$S_F(x) = \lim_{\varepsilon\to 0} \frac{-i}{(2\pi)^4} \int d^4q\, e^{-iq_\mu x_\mu} \frac{i\gamma q - M}{q^2+M^2-i\varepsilon}. \tag{3}$$

The wave amplitudes for the proton and the antiproton can be taken respectively as†

$$\begin{aligned}\psi(x) &= u(p)\exp(ip_\mu x_\mu),\\ \psi(x') &= \bar{v}(p')\exp(ip'_\mu x'_\mu),\end{aligned} \tag{4}$$

and the wave amplitude of the π^0 as

$$\phi(x) = (2\varkappa_0)^{-\frac{1}{2}}\exp(-i\varkappa_\mu x_\mu). \tag{5}$$

Thus,

$$\begin{aligned}S_2 = -i(G^2/2)(2\pi)^4\delta^4(p_\mu+p'_\mu-\varkappa_\mu-\varkappa'_\mu)(\varkappa_0\varkappa'_0)^{-\frac{1}{2}}\\ \times\bar{v}(p')\gamma_5\frac{i\gamma(p-\varkappa)-M}{(p-\varkappa)^2+M^2}\gamma_5 u(p).\end{aligned} \tag{6}$$

We shall frequently use the following relations:

$$(i\gamma p+M)u(p) = \bar{v}(p')(i\gamma p'-M) = 0; \tag{7}$$

$$p^2+M^2 = p'^2+M^2 = \varkappa^2+\mu^2 = 0. \tag{8}$$

Further we shall completely neglect the rest mass of the meson compared with its energy so that $\varkappa_0 \approx |\mathbf{\varkappa}|$.

Then we have

$$S_2 = (iG^2/2)(\varkappa_0\varkappa'_0)^{-\frac{1}{2}}M_2(2\pi)^4\delta^4(p_\mu+p'_\mu-\varkappa_\mu-\varkappa'_\mu), \tag{9}$$

where

$$M_2 = \bar{v}(p')[(i\gamma\varkappa)/(2p\varkappa)]u(p) \tag{10}$$

and

$$(p\varkappa) = \mathbf{p}\cdot\mathbf{\varkappa}-p_0\varkappa_0. \tag{11}$$

In the C.-M.-system, we have

$$\mathbf{p}' = -\mathbf{p}, \qquad \mathbf{\varkappa}' = -\mathbf{\varkappa}; \quad p_0 = p'_0 = \varkappa_0 = \varkappa'_0. \tag{12}$$

We then find from (10) and (11) that M_2 has a sharp maximum when $\theta = 0$, θ being the angle between $\mathbf{p}$ and $\mathbf{\varkappa}$. Thus most of the contribution comes from values of $\mathbf{\varkappa}$ lying within a narrow cone around $\mathbf{p}$, in which case (from (12)) $\mathbf{\varkappa}'$ lies within a narrow cone around $\mathbf{p}'$.

The differential cross section is given by the Møller invariant formula [5]):

$$\begin{aligned}d\sigma_2 = (p_0/2p)(G^2/2)^2|M_2|^2(2\pi)^4\delta^4(p_\mu+p'_\mu-\varkappa_\mu-\varkappa'_\mu)\\ \times(2\pi)^{-6}(d^3\mathbf{\varkappa}/\varkappa_0)(d^3\mathbf{\varkappa}'/\varkappa'_0).\end{aligned} \tag{13}$$

The integrations over $d^3\mathbf{\varkappa}'$ and $d\varkappa_0$ by using the δ-function gives, with the help of (12),

† The momenta of the proton and the anti-proton are taken to be p_μ and p'_μ respectively. The corresponding negative energy proton has 4-momentum $-p'_\mu$. Further, all normalizations are made within unit volume.

$$\int \delta(p_0+p_0'-\varkappa_0-\varkappa_0')\,\mathrm{d}\varkappa_0 = \mathrm{d}\varkappa_0/\mathrm{d}(\varkappa_0+\varkappa_0') = \tfrac{1}{2}. \tag{14}$$

We must also average over the spin directions of the fermions in the initial state, for which purpose we use the following formulae:

$$\langle u_\alpha(p)\bar{u}_\beta(p)\rangle_{\mathrm{Av}} = -(1/4p_0)(i\gamma p - M)_{\alpha\beta}; \tag{15a}$$

$$\langle v_\alpha(p')\bar{v}_\beta(p')\rangle_{\mathrm{Av}} = -(1/4p_0')(i\gamma p' + M)_{\alpha\beta}. \tag{15b}$$

Thus we have

$$\begin{aligned}\langle|M_2|^2\rangle_{\mathrm{Av}} &= -(1/16)(p_0p_0')(2p\varkappa)^{-2}\,\mathrm{tr}[(i\gamma p - M)(\gamma\varkappa)(i\gamma p' + M)(\gamma\varkappa)] \\ &= (1/8)(p_0p_0')[(p'\varkappa)/(p\varkappa)].\end{aligned} \tag{16}$$

Collecting all the results from (12) and (16) we have the total cross section as

$$\sigma_2 = (1/32)(p_0p)(G^2/4\pi)^2\int \mathrm{d}\Omega_\varkappa[(p_0+p\cos\theta)/(p_0-p\cos\theta)], \tag{17}$$

where θ is the angle between $\mathbf{p}$ and $\boldsymbol{\varkappa}$. Integration over θ gives

$$\sigma_2 = (\pi/16)(p_0p)^{-1}(G^2/4\pi)^2[2(p_0/p)\ln[(p_0+p)/(p_0-p)]-2]. \tag{18}$$

So far everything is exact, except for neglecting μ^2. We now consider the extreme relativistic case in which $p_0^2 \gg M^2$. Thus we have

$$\sigma_2 \approx (\pi/4)p_0^{-2}(G^2/4\pi)^2\ln(2p_0/M).$$

This last expression has to be multiplied by a factor 2, since we get an equal contribution to the cross section by interchanging the roles of the 2 fermions. Thus the complete cross section is

$$\sigma_2 = (\pi/2)p_0^{-2}(G^2/4\pi)^2\ln(2p_0/M). \tag{19}$$

For later purposes it will be useful to integrate (17) over values of $\boldsymbol{\varkappa}$ lying within a narrow cone around $\mathbf{p}$. If δ is the maximum angle allowed, we have

$$\begin{aligned}I(\delta) &= \int_0^\delta \sin\theta\,\mathrm{d}\theta(p_0+p\cos\theta)/(p_0-p\cos\theta) \\ &\approx \int_0^\delta \theta\,\mathrm{d}\theta\,(2p_0)^2[M^2+p\theta^2]^{-1} \approx 4\ln(p_0\delta/M).\end{aligned} \tag{19a}$$

The corresponding total cross section is

$$\sigma_2(\delta) = (\pi/2)p_0^{-2}(G^2/4\pi)^2\ln(p_0\delta/M). \tag{20}$$

Thus the effect is just to replace the factor $\ln(2p_0/M)$ in (19) by $\ln(p_0\delta/M)$.

3. n-π^0 Annihilation

Following Dyson's method [4]), the S-matrix element for this case is given by

$$S_n = G^n \int \mathrm{d}x_1 \mathrm{d}x_2 \ldots \mathrm{d}x_n \phi(x_1)\phi(x_2)\ldots\phi(x_n)$$
$$\times \bar{\psi}(x_n)\gamma_5 S_F(x_{n-1}-x_n)\gamma_5 \ldots S_F(x_1-x_2)\gamma_5\psi(x_1). \quad (21)$$

We denote the 4-momenta of the various π^0's by the four-vectors $\varkappa_n$ (components $\boldsymbol{\varkappa}_n$, $i\varkappa_{0,n}$). The 4-momenta of the proton and the antiproton are p and p' respectively. Using the momentum representation (3), (4) and (5), we obtain from (21)

$$S_n = (-i)^{n-1}(G^2/2)^{n/2}$$
$$(\varkappa_{0,1}\varkappa_{0,2},\ldots\varkappa_{0,n})^{-\frac{1}{2}} M_n (2\pi)^4 \delta^4(p+p'-\varkappa_1-\varkappa_2-\ldots\varkappa_n), \quad (22)$$

where

$$M_n = \frac{\overline{v}(p')\gamma_5(i\gamma p_{n-1}-M)\gamma_5\ldots(i\gamma p_1-M)\gamma_5 u(p)}{(p_{n-1}^2+M^2)\ldots(p_1^2+M^2)}. \quad (23)$$

The quantities $p_1,\ldots p_{n-1}$ are the propagation vectors of the internal lines and are as defined by equation (49) of A. Following Gupta, we assume that the r mesons $\boldsymbol{\varkappa}_1, \boldsymbol{\varkappa}_2, \ldots \boldsymbol{\varkappa}_r$ are emitted within a narrow cone of angle δ around the vector $\mathbf{p}$ and the remaining $(n-r)$ mesons $\boldsymbol{\varkappa}_{r+1},\ldots\boldsymbol{\varkappa}_n$ are emitted within a narrow cone of angle δ around the vector $\mathbf{p}'$. If we neglect $o(\delta^2)$ we have approximately in the C.M. system ($\mathbf{p} = -\mathbf{p}'$)

$$\varkappa_{0,1}+\varkappa_{0,2}+\ldots+\varkappa_{0,r} = \varkappa_{0,r+1}+\varkappa_{0,r+2}\ldots+\varkappa_{0,n} = p_0. \quad (24)$$

Then M_n may be written as

$$M_n = \frac{\overline{v}(p')\gamma_5[i\gamma(\varkappa_n-p')-M]\ldots\gamma_5[i\gamma(\varkappa_n+\ldots+\varkappa_{r+2}-p')-M]}{[(\varkappa_n-p')^2+M^2]\ldots[(\varkappa_n+\ldots+\varkappa_{r+2}-p')^2+M^2]} \quad (25)$$
$$\times \frac{\gamma_5[i\gamma(p-\varkappa_1-\varkappa_2\ldots-\varkappa_r)-M]\gamma_5\ldots[i\gamma(p-\varkappa_1)-M]\gamma_5 u(p)}{[(p-\varkappa_1-\varkappa_2\ldots-\varkappa_r)^2+M^2)]\ldots[(p-\varkappa_1)^2+M^2]}.$$

As in A, we assume that $\varkappa_{0,r}$ and $\varkappa_{0,r+1}$ in (25) are large and all the other $\varkappa_0$'s are small. We now introduce the following notations:

$$a_i = -(2p\varkappa_i), \quad i = 1, 2, \ldots r;$$
$$a'_j = -(2p'\varkappa_j), \quad j = r+1, \ldots n. \quad (26)$$

The denominators in (25) can then be simplified as in A and the results expressed in terms of the quantities a_i and a'_j. The numerators, however, present some additional difficulties on account of

the peculiar anti-commuting nature of the γ_5-matrix. The numerators can be simplified as follows:

$$[i\gamma(p-\varkappa_1-\varkappa_2)-M]\gamma_5[i\gamma(p-\varkappa_1)-M]\gamma_5 u(p)$$
$$= [i\gamma(p-\varkappa_1-\varkappa_2)-M](i\gamma\varkappa_1)u(p) \approx (-2p\varkappa_1)u(p), \quad (27)$$

since we may neglect terms like $(\varkappa_1\varkappa_2)$ compared with $(p\varkappa_1)$. This sort of reduction can be carried out successively, beginning from the *left* in the first group of terms in (25), and from the *right* in the second group of terms. Assuming that both $(r-1)$ and $(n-r-1)$ are *even*, and using (26) and (27) we have

$$\bar{v}(p')\gamma_5[i\gamma(\varkappa_n-p')-M]\ldots\gamma_5[i\gamma(\varkappa_n+\ldots\varkappa_{r+2}-p')-M]$$
$$\approx \bar{v}(p')a'_n(a'_n+a'_{n-1}+a'_{n-2})\ldots(a'_n+\ldots+a'_{r+3}); \quad (28)$$

$$\gamma_5[i\gamma(p-\varkappa_1-\ldots-\varkappa_{r-1}-\varkappa_r)-M]\gamma_5\ldots[i\gamma(p-\varkappa_1)-M]\gamma_5 u(p)$$
$$\approx a_1(a_1+a_2+a_3)\ldots(a_1+\ldots+a_{r-2})(i\gamma\varkappa_r)u(p). \quad (29)$$

Using the equations (26)—(29) we can finally write M_n as

$$M_n = [\bar{v}(p')(i\gamma\varkappa_r)u(p)/(-2p\varkappa_r)]$$
$$\times\left[\frac{a'_n(a'_n+a'_{n-1}+a'_{n-2})\ldots(a'_n+\ldots+a'_{r+3})}{a'_n(a'_n+a'_{n-1})\ldots(a'_n+\ldots+a'_{r+2})}\right] \quad (30)$$
$$\times\left[\frac{a_1(a_1+a_2+a_3)\ldots(a_1+\ldots+a_{r-2})}{a_1(a_1+a_2)\ldots(a_1+\ldots+a_{r-1})}\right].$$

We have now to sum (30) over all possible terms like (30) which are obtained by interchanging the roles of $\varkappa_n,\ldots\varkappa_{r+2}$ in the first group of terms and those of $\varkappa_1,\ldots\varkappa_{r-1}$ in the second group of terms. It is there that we experience a difficulty in so far as we cannot use the identity (61) of A, which was so vital for the derivation of an elegantly simple formula obtained by Gupta. We shall try to avoid this difficulty by obtaining a *lower limit* on the cross section σ_n. For this latter purpose we proceed as follows.

We first note that the quantities a_i and a'_j are *all positive* as may be seen immediately from (26). Thus we shall be making things unfavourable for ourselves (from the point of view of getting a large cross section) if we ignore some of the a's in the numerators of (30). Thus

$$a'_n(a'_n+a'_{n-1}+a'_{n-2})\ldots(a'_n+\ldots+a'_{r+3})$$
$$> a'_n(a'_{n-1}+a'_{n-2})(a'_{n-3}+a'_{n-4})\ldots(a'_{r+4}+a'_{r+3}). \quad (31)$$

An exactly similar inequality holds for the second numerator.

The inequality (31) should still hold quite strongly even if we

replace a'_n by $(a'_n+a'_{r+2})$, since all these quantities are of the same order of magnitude. Next, we use the inequality

$$a+b \geqq 2(ab)^{\frac{1}{2}}. \tag{32}$$

Thus we obtain the still stronger inequality

$$a'_n(a'_n+a'_{n-1}+a'_{n-2}) \ldots (a'_n+ \ldots a'_{r+3}) > (2)^{\frac{n-r-1}{2}} [a'_n a'_{n-1} \ldots a'_{r+2}]^{\frac{1}{2}}. \tag{33}$$

Similarly we have the other inequality

$$a_1(a_1+a_2+a_3) \ldots (a_1+a_2+ \ldots a_{r-2}) > (2)^{\frac{r-1}{2}} [a_1 a_2 \ldots a_{r-1}]^{\frac{1}{2}}. \tag{34}$$

The advantages of (33) and (34) are that their right- hand sides are completely symmetrical in the various arguments, so that if we replace the respective numerators in (30) by these latter quantities, we can use the identity (61) of A.

If now K'_n denotes the sum of all possible terms M_n obtained by permuting the arguments in all possible ways, we find by using (33), (34) and (A61) that $|K'_n| > |K_n|$ where

$$K_n=(2)^{\frac{n-2}{2}} (a_1 \ldots a_{r-1} a'_{r+2} \ldots a'_n)^{-\frac{1}{2}} \times [\bar{v}(p')(i\gamma\varkappa_r)u(p)/(-2p\varkappa_r)]. \tag{35}$$

The differential cross section in the C.M. system is now given by [5])

$$\mathrm{d}\sigma_n = (p_0/2p)(G^2/2)^n |K'_n|^2 (2\pi)^4 \delta^4(p+p'-\varkappa_1- \ldots -\varkappa_n) \times \prod_{i=1}^{n} [\mathrm{d}^3\boldsymbol{\varkappa}_i/(2\pi)^3 \varkappa_{0,i}]. \tag{36}$$

Replacing K'_n by K_n in (36) we get a lower limit on σ_n which we shall denote by σ_n^0. Integrating (36) over $\mathrm{d}^3\boldsymbol{\varkappa}_{r+1}$ and $\mathrm{d}\varkappa_{0,r}$ by means of the δ-function, we obtain with the help of (14) and (24)

$$\sigma_n^0 = (p_0/4p)(G^2/2)^n (2\pi)^{-3n+4} \int \mathrm{d}\Omega_r(\varkappa_{0,r}/\varkappa_{0,r+1}) \times \prod_{i=1}^{r-1} (\mathrm{d}^3\boldsymbol{\varkappa}_i/\varkappa_{0,i}) \prod_{j=r+2}^{n} (\mathrm{d}^3\boldsymbol{\varkappa}_j/\varkappa_{0,j}) \langle K_n^* K_n \rangle_{\mathrm{Av}}. \tag{37}$$

Now using equations (15), we have from (35)

$$\langle K_n^* K_n \rangle_{\mathrm{Av}} = (1/8p_0^2)[(p'\varkappa_r)/(p\varkappa_r)] 2^{n-2} [a_1 \ldots a_{r-1} a'_{r+2} \ldots a'_n]^{-1}. \tag{38}$$

Substituting (38) and (26) in (37), the integrations can be carried out in a straightforard manner. We take $\varkappa_{0,r}$ and $\varkappa_{0,r+1}$ as of the order of p_0. As regards the other $\varkappa'_0$s we shall take their upper limits of integration to be $\varkappa_{\max}$ which should preferably be somewhat smaller than p_0 since we have regarded these $\varkappa'_0$s as small compared with p_0. We obtain after some reduction

$$\sigma_n^0 = (1/32p_0^2)(G^2/4\pi)^n(2\pi)^{-n+3}\int d\varkappa_{0,1}\cdots\int d\varkappa_{0,r-1}\int d\varkappa_{0,r+2}\cdots\int d\varkappa_{0,n}$$
$$\times\int_0^\delta \theta_1\,d\theta_1\cdots\int_0^\delta \theta_r\,d\theta_r\int_0^\delta \theta_{r+2}\,d\theta_{r+2}\cdots\int_0^\delta \theta_n\,d\theta_n$$
$$\times\left[\sum_{i=1}^{r-1}\frac{\varkappa_{0,i}}{(-p\varkappa_i)}\right]\left[\prod_{j=r+2}^{n}\frac{\varkappa_{0,j}}{(-p'\varkappa_j)}\right]\left(\frac{p_0+p\cos\theta_r}{p_0-p\cos\theta_r}\right).$$

Carrying out the various integrations we have

$$\sigma_n^0 = (1/4p_0^2)(G^2/4\pi)^n\pi^{-n+3}(\varkappa_{\max}/p_0)^{n-2}\times[\ln(p_0\delta/M)]^{n-1}.$$

As in A, this expression has to be multiplied by the factor $2^{n-2}/(n-2)!$ to take account of the indistinguishability of the mesons and also by a factor 2 to allow for the interchange of the two fermions. Thus we have finally

$$\sigma_n^0 = \frac{1}{(n-2)!}\left(\frac{2}{\pi}\right)^{n-3}\left(\frac{1}{p_0^2}\right)\left(\frac{G^2}{4\pi}\right)^n\left(\frac{\varkappa_{\max}}{p_0}\right)^{n-2}\left(\ln\frac{p_0\delta}{M}\right)^{n-1}. \tag{39}$$

This is only a lower limit but it reduces exactly to (20) when we put $n = 2$. This is not fortuitous if we remember that for $n = 2$ the inequalities (33) and (34) reduce to identities! Dividing (39) by (20), we have

$$\sigma_n^0 = \frac{\sigma_2}{(n-2)!}\left[\left(\frac{G^2}{2\pi^2}\right)\left(\frac{\varkappa_{\max}}{p_0}\right)\left(\ln\frac{p_0\delta}{M}\right)\right]^{n-2}. \tag{40}$$

4. Conclusion

We are now in a position to compare our expression (40) for σ_n^0 with the corresponding one of Gupta. We note first that the dependence of σ_n^0 on the logarithmic factor $\ln(p_0\delta/M)$ (involving angular integration) is the same in both cases. However, the dependence on the factor representing the energy limits on the various quanta is quite different. In Gupta's case, this second factor is also logarithmic in $\varkappa_{\max}$, but in the present one it is *linear*, as may be seen from (40). This result is independent of the inequalitites (33) and (34) which were used to simplify the calculations. The reason for this difference becomes clear when one compares the structures of the S-matrix elements in the two cases. One finds that in the present case the numerator of the matrix element involves factors like $(p\varkappa_r)$ while in Gupta's case the corresponding factors were of the form $\mathbf{p}\cdot\mathbf{e}_r$. Thus our matrix element has additional factors involving meson momenta in the

numerator, as a result of which the dependence of σ_n on the magnitudes of these momenta is stronger than in A.

As regards the magnitude of σ_n in the present case, we take the incident anti-proton energy (lab. system) to be $E' \approx 10^{15}$eV (as in A), corresponding to a C.M. energy E given by

$$E = (2E'Mc^2)^{\frac{1}{2}} \approx 10^{12}\text{eV} \quad (\text{since } Mc^2 \approx 10^9\,\text{eV}).$$

The value of δ may be taken to be $\approx 1/10$ since the mesons are assumed to be confined to narrow cones about $\mathbf{p}$ and $\mathbf{p}'$. Further, $(G^2/4\pi) \approx 16$. Finally, $\varkappa_{\max}$ may be estimated roughly from the consideration that all the mesons except $\varkappa_r$ and $\varkappa_{r+1}$ have smaller energies than the latter, so that their upper limits should be somewhat smaller than p_0. For purposes of rough estimation it seems very reasonable to take $\varkappa_{\max} \approx (1/10)p_0$ (smaller than p_0 by an order of magnitude). Substituting the above numerical estimates in (40), we obtain

$$\sigma_n > \sigma_n^0 \approx [\sigma_2/(n-2)!]5^{n-2}. \tag{41}$$

Eq. (41) may be compared with the corresponding equation (110) of A. We note that in spite of making the most unfavourable estimates in the present case, we obtain an additional factor $\approx 5^{n-2}$ as compared with A. On account of this last factor, σ_n^0 increases rapidly up to a fairly large value of n until the other factor $(n-2)!$ takes over. Using Stirling's approximation, the value of n at which $\sigma_n^0 \approx \sigma_2$ is roughly given by

$$5^{n-2} \approx (n-2)^{n-2}e^{-n+2}, \tag{42}$$

i.e.,
$$n \approx 15.$$

Thus the cross section begins to decrease below σ_2 only for the production of more than about 15 π^0's. This already accounts for about $2\times 15 = 30$ photons. It is by no means claimed that this figure should be taken seriously since the approximations involved are considerable, but the order of magnitude of n may be emphasized, particularly because we have taken a very safe lower limit on σ_n.

As regards the bearing of this result on the Schein event, it may be remarked that though the order of magnitude of the cross section is quite favourable for the production of a good number of photons, the mechanism proposed here does not ensure the absence of charged pion production. In fact, on the basis of charge independence one can estimate that the probability for charged pion

production will be quite considerable as well. Thus unless similar cosmic ray events involving charged pions are observed with at least equal frequency, it is doubtful that this mechanism will prove an explanation of the Schein event.

I am indebted to Professors P. S. Gill (Aligarh University) and R. C. Majumdar (Delhi University) for useful discussions on the results obtain in this paper.

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THE SPIN OF THE 2.9 MeV STATE OF Be^8

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Abstract: The spin of the 2.9 MeV state of Be^8 has been determined by an angular correlation measurement between the direction of emission of the 14.8 MeV γ-rays and the corresponding α-particles arising from the state formed in the reaction $Li^7(p\gamma)Be^{8*}(\alpha)He^4$. It is found to be 2 and the results also show that the γ-transition from the 17.6 MeV 1^+ state to the 2^+ state is a mixture of M1 and E2 radiations.

Although it has been generally accepted on theoretical grounds that the spin of the 2.9 MeV state in Be^8 is 2^+, until recently the main experimental evidence came from studies of the shape of the alpha-particle spectra observed in the photodisintegration of C^{12} (Telegdi [1]) and in the reaction $B^{10}(d, \alpha)Be^8$ studied in this laboratory (Treacy [2])). Geer *et al.* [3]) have provided further confirmation by measuring the alpha-alpha angular correlations in $B^{11}(p, \alpha\alpha)He^4$ but, in the course of a programme of work in Canberra on the energy levels of Be^8, the spin has been determined by measuring the correlation between the directions of emission of alpha-particles and gamma-radiation at the 440 keV resonance in the following reaction:

$$Li^7+p \rightarrow Be^{8**} \rightarrow Be^{8*}+\gamma \rightarrow 2\alpha+\gamma.$$

Separated Li^7 targets of 20 $\mu g/cm^2$ superficial weight, electrolytically deposited on a copper backing, were bombarded with a beam of 450 keV protons. The alpha-particles and gamma-rays were both observed in the plane perpendicular to the beam. The gamma-rays, whose energies ranged from 10 to 16 MeV, were detected with a scintillation counter which was capable of rotation about the target while the alpha-particles were detected by a thin NaI crystal situated in the vacuum at 12 cm from the target and connected by a light pipe to the photomultiplier. To reach this crystal the radiation from the target passed through a magnetic field of approximately 7000 Gauss and a system of collimators which

excluded both light and scattered protons. Foils could not be used in this case since the energy of the alpha-particles concerned was only 1 or 2 MeV. Both photomultipliers were magnetically shielded so that no change in pulse height resulted from their presence in the fringing field of the magnet. Pulses from each counter were limited in size and passed to a coincidence unit the output of which was used to "gate" the proportional pulses from the alpha-counter so that only the spectrum of alpha-particles in coincidence with a gamma-ray was recorded in the multichannel analyser.

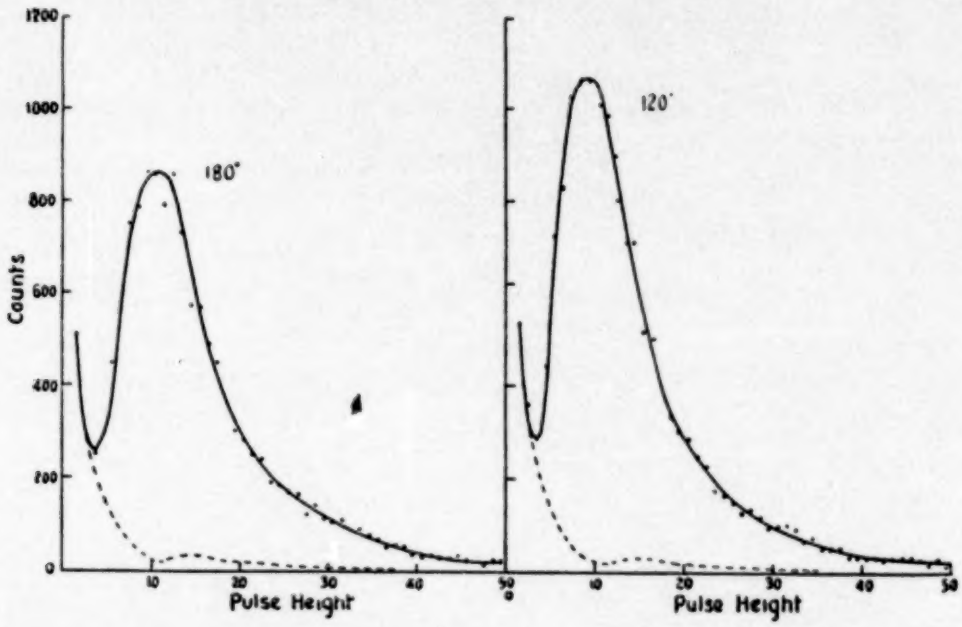

Fig. 1

The alpha-particle spectra taken with angles of 180° and 120° between the counters for a fixed number of gamma-rays are shown in fig. 1 and represent the sum of a large number of spectra taken alternately between the two angles. The background spectra due to random coincidences are shown by the dotted lines.

The pulse height scale was calibrated using the alpha-particles from ThC. Upon applying this calibration and correcting for the angle of emission of the gamma-ray, the two spectra (corrected for random background) can be shown on an energy scale corresponding to the excitation in Be^8, as in fig. 2.

The possibilities for the value of the spin of the 2.9 MeV state are either 0 or 2. If the spin were zero there would be no correlation between the directions of emission of the alpha-particle and the gamma-ray, and the spectra taken at the two angles would be iden-

tical. When the spin is 2 the correlation depends on whether the gamma-radiation is pure magnetic dipole, pure electric quadrupole, or a mixture of both. The ratio of the intensities at 120° and 180° is 2.1 for pure magnetic dipole and 0.25 for pure electric quadrupole. The experimental ratio obtained from the peak of the curves in fig. 2 is ≈ 1.2 indicating that the spin of the 2.9 MeV state of Be^8 is 2.

Further interpretation of these curves is restricted for two reasons. Firstly, although the geometric solid angle of the gamma counter was small (about $4\pi/125$), it was effectively greater since the mag-

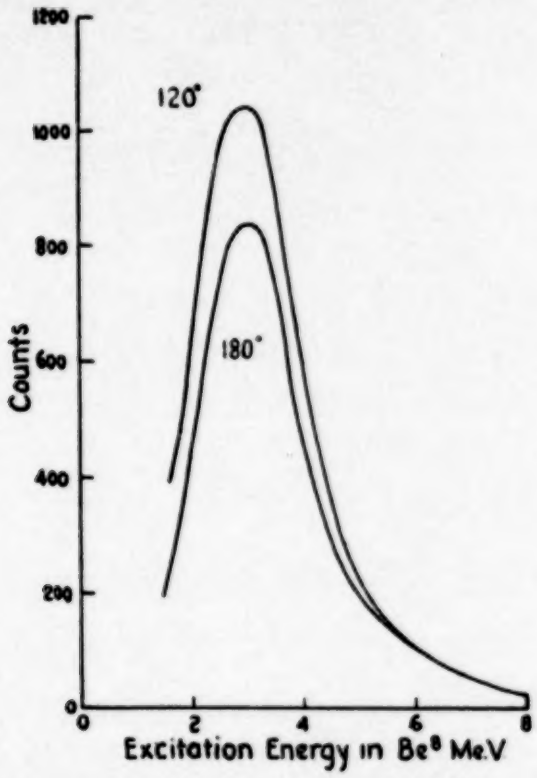

Fig. 2

netic shield surrounding the counter and the magnet itself provided much high density scattering material close to the crystal. Secondly, the solid angle subtended by the target at each point on the alpha-detecting crystal was not constant during the experiment: because of the system of collimation a small change in the position of the beam produced a change in the solid angle. These effects both lead to an indeterminate lessening of the anisotropy of the correlation. The curves of fig. 2 do indicate, however, that the transition to the 2.9 MeV state is not pure magnetic dipole, but has an admixture of electric quadrupole radiation in it.

Low intensity transitions to other states (Inall and Boyle [4])) would not be observed in this experiment as the resolution of the alpha-particle detector at low energies was poor, being about 25 %.

This work was part of the research programme in nuclear physics of this laboratory under Professor E. W. Titterton.

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OBSERVATION OF THE LARGE COSMIC RAY INCREASE OF FEBRUARY 23, 1956 IN THE U.S.S.R.

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Cape Schmidt Cosmic Ray Station

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The latest considerable increase (outburst) of cosmic ray intensity which took place on February 23, 1956 was also the largest variation recorded heretofore. This increase was observed at five cosmic ray stations in the U.S.S.R. (see table 1).

TABLE 1

Station	Geomagnetic latitude, N	Geographic longitude, E	Altitude above sea level, metres	Thickness of roof, g/cm²
Moscow	52.0	37.3	203	20
Sverdlovsk	48.5	61.0	290	40
Tbilisi	37.1	44.7	404	20
Yakutsk	51.0	129.0	100	108 (10)†
Cape Schmidt	63.0	179.5	6	20

† Thickness of the roof above the counter telescope in Yakutsk. The other data in the fifth column refer to ionization chambers.

The cosmic ray intensity I at these stations was recorded by continuously running automatic spherical ionization chambers in which the current corresponding to the mean value of I was compensated. The chambers had a volume of 950 l (ACK—I) and 50 l (ACK—2); they were shielded by a 12 cm layer of lead and filled with very pure argon to a high pressure.

During a one hour run the accuracy of measurement of I was 0.5 % for the small chamber and 0.1 % for the large one [1]).

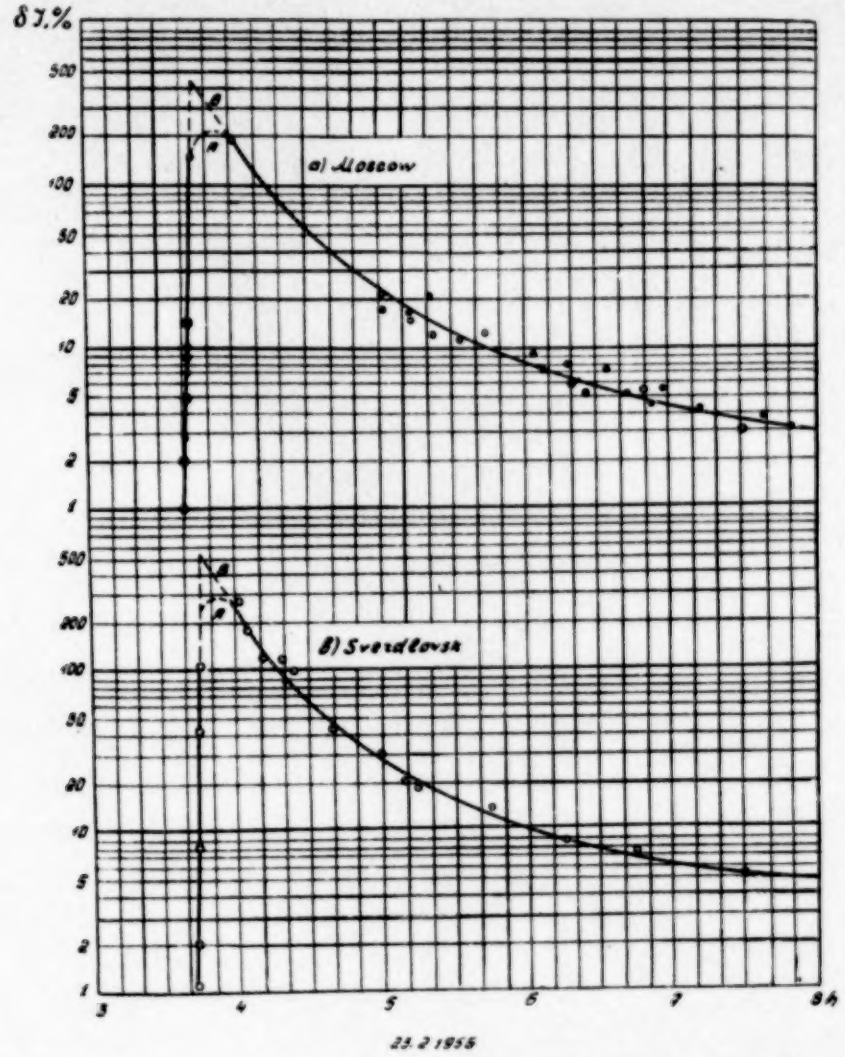
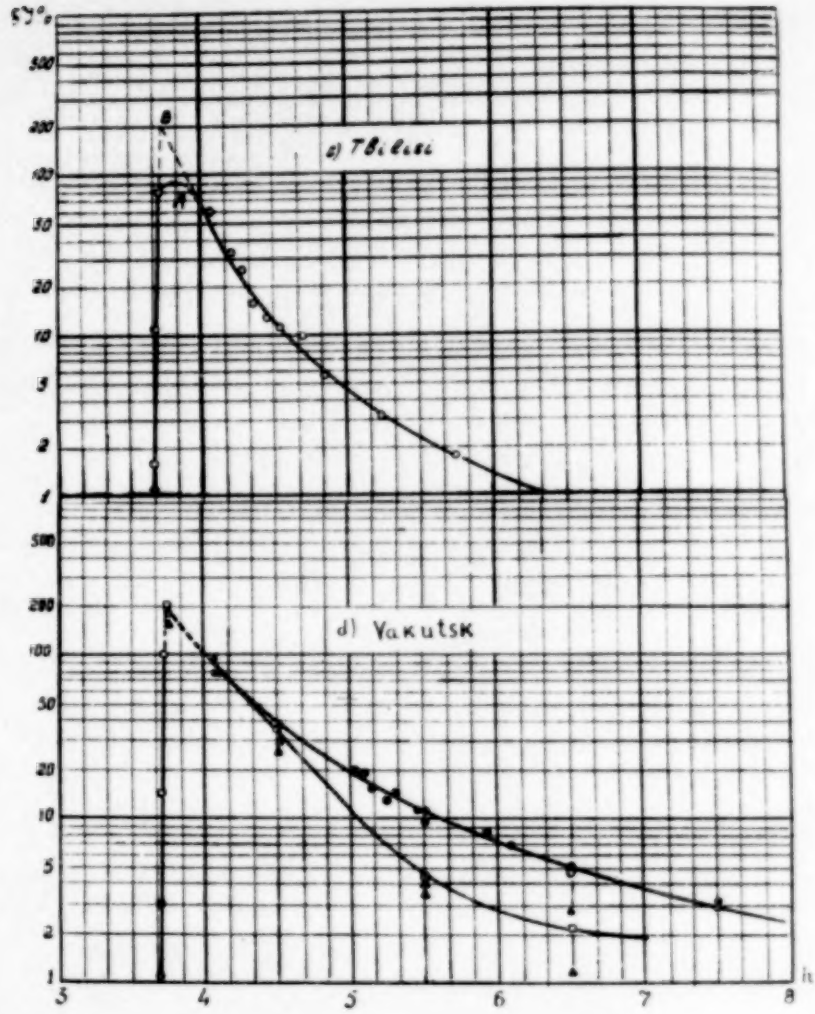


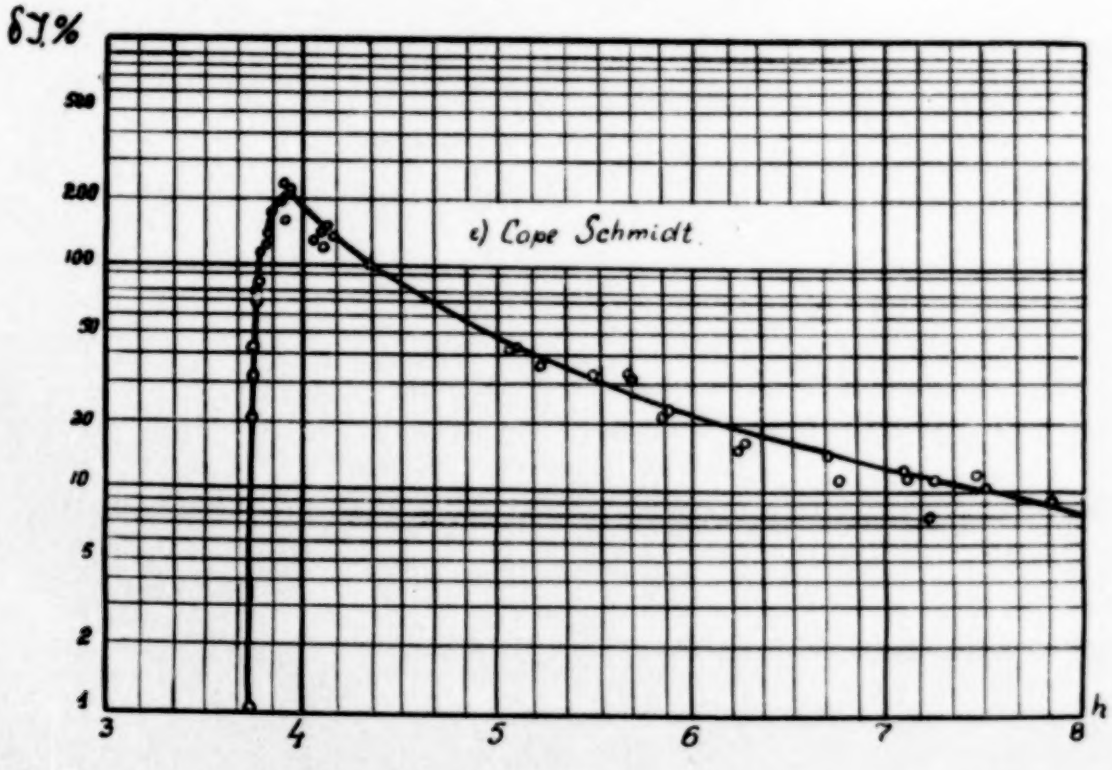
Fig. 1. Variation of intensity of the cosmic ray hard component during the increase of February 23, 1956

a) in Moscow, b) in Sverdlovsk, c) in Tbilisi, d) in Yakutsk, e) at Cape Schmidt.

Symbols:

- ● large chamber
- ○ small chamber
- □ counter telescope, vertical direction
- △ counter telescope directed south
- ▲ counter telescope directed north


δJ%
500
200
100
50
20
10
5
2
1
c) Tbilisi
d) Yakutsk
3
4
5
6
7
8
h


δJ%
500
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c) Cape Schmidt
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23 II 1956.

In Moscow the cosmic ray increase was recorded on February 23, 1956 at 3.40 Greenwich time (6.10 local time). The measurements were carried out in two closely located chambers ACK—1 and ACK—2.

A comparatively slow increase of I in the big chamber commenced at 3.38 and equalled 2 % within 2 minutes; at 3.43 it reached 13 %. After 3.43 the increase of I was so rapid that it was practically impossible to follow in detail the readings of the meter. It can only be said that at 3.45 the cosmic ray intensity exceeded the normal value by at least 200 %. At the same time an increase of 180 % was observed in the small chamber.

The experimental results are illustrated in fig. 1a, in which the data are plotted on a semi-logarithmic scale. On this scale the plot of δI is almost a straight line and this signifies an exponential law of variation of intensity. The ascending part of the curve can be expressed by exp $(t/0.02)$, where t is in hours. This means that the rise is considerably more rapid than that observed during the last solar flare on November 19, 1949, which could be expressed by the formula [2]) exp $(t/0.07)$. The intensity decreased at a slower rate. At first I varied as exp $(-t/0.45)$ and at the end of the outburst as exp $(-t/2.2)$ †. The increase could be followed up to 12 o'clock Greenwich time. In about 8.5 hours after the start of the increase the fluctuations of I did not exceed the mean Moscow value.

The lowest possible values of the intensity increase near the maximum is given by the dotted curve A which shows that the maximum value of δI was approximately 200 %. Dotted line B corresponds to the most probable extrapolation, the maximum value of δI in this case being 400 %.

In Sverdlovsk a similar increase was observed in a ACK—2 chamber. Due to lower sensitivity of the apparatus the conditions for observation were more favourable. The increase in Sverdlovsk also began at 3.40 Greenwich time (7.44 local time) and within several minutes reached a probable value of 500 % (see fig 1b).

In Tbilisi at 3.40 (6.40 local time) an increase somewhat larger than 100 % was observed in a similar apparatus (fig. 1c).

In Yakutsk the cosmic ray increase was recorded by two chambers, ACK—1 and C—2 (volume 19 l) and by continuously operated automatic counter telescopes shielded with 8 cm lead. The telescopes

† On November 19, 1949 the intensity returned to its normal value according to the law [2]) exp $(-t/3)$.

recorded particles moving in the vertical direction and at angles of 30° in the southern and northern directions. The thickness of the roof above the counter telescope was 10 g/cm². The increase started in Yakutsk at 3.42 (12.12 local time). As can be seen from fig. 1d the increase δI recorded by the counter telescope reached 200 %, the same as that recorded by the ionization chambers. For vertically moving particles the increase was 200 %, for particles moving from the south 190 % and for particles moving from the north it was somewhat less, about 165 %. It can also be seen from fig. 1d that the rate of decrease of the intensity was slower in the chamber than in the telescope [3]).

No change in the shower frequency was recorded during the increase of February 23, 1956 by any of the two automatic experimental arrangements located under a light roof (2 g/cm²) in Yakutsk. The arrangements were sensitive to extensive atmospheric showers possessing a particle density between 25 and 50 particles per m² and created by primary particles of about 10^{14} eV energy.

At Cape Schmidt the cosmic ray outburst was recorded by a ACK—2 chamber shielded with 12 cm Pb. The increase began at 3.42 Greenwich time (15.42 local time) and within 10 minutes attained a value of 200 %. The subsequent slow decrease lasted about 8 hours. In distinction to the experimental data obtained at the other stations those obtained at the Cape Schmidt station permit one to follow the course of the intensity curve near the maximum (see fig. 1e). As can be seen from the graph the true values of the intensities measured in Moscow, Sverdlovsk and Tbilisi lie between the extreme extrapolated values A and B (apparently somewhat closer to B).

We were unable to establish a connection between the intensity increase and chromosphere eruptions on the sun. However, on the northwestern edge of the visible disk of the sun a very active region with several groups of spots surrounded by bright flocules could be observed. Over 20 chromosphere flares, eruptive protuberances and filaments were observed in the region of these groups of spots. The solar corona in this region was strongly excited and a bright glow of the yellow corona line was observed which testifies to a high activity in this region of the sun. On February 23, at 4.25 Greenwich time a large chromosphere flare (importance rating 2) was observed in the vicinity of the largest group of spots.

Only a small increase of radiowave absorption in the D layer

was recorded by the ionosphere stations in Moscow on February 23. At the Yakutsk ionosphere station of the same day complete absorption of radiowaves in the 2—7 megacycle band was observed at 4 o'clock Greenwich time. Thus, a strong ionosphere perturbation (Deelinger effect) was observed on that side of the earth which faced the sun at the beginning of the intensity increase. This perturbation caused a radio fadeout on the territory of the Yakutsk Republic within a radius up to 4000 km from the city of Yakutsk. Only solar chromosphere eruptions of unheard intensity can account for such a large increase of absorption.

As a rule such chromosphere eruptions are accompanied by the emission of solar corpuscular rays moving with a velocity of about 10^8 cm/sec; within one or two days these particles may reach the earth (if the eruption occurred near the central solar meridian) and cause a magnetic storm accompanied by a considerable decrease of I at all latitudes, including the equator. This is exactly what happened during the chromosphere eruptions of February 28, 1942 and July 25, 1946 which took place near the central meridian. During the outbursts of November 19, 1949 (at 70° west of the central meridian) and February 23, 1956 no such phenomenon was observed. These facts mean that whereas the corpuscular rays are emitted from the sun in a direction close to the radial direction, cosmic rays are emitted by the sun in a comparatively broad solid angle.

The last cosmic ray increase differs from the preceding ones. Thus, the increase of November 19, 1949 occurred about 30—60 minutes later in Europe than on the American continent [4]). According to our data the intensity increase was almost simultaneous (to within a few minutes) over a very large territory (Moscow-Yakutsk).

The longitude dependence of the cosmic ray intensity during the last outburst seems to agree with the results obtained in ref. [5]); according to these results the largest increase of I due to additional cosmic ray particles approaching the earth from the sun must be several times higher in some certain zone † than in the remaining regions.

The intensity increase of February 23, 1956 was extraordinarily large compared to other outbursts. Thus, during the November 19,

† This zone was located in America during the increase of November 19, 1949 and in Europe during the February 23, 1956 increase.

1949 increase which was the greatest of the four known cosmic ray outbursts the hard component intensity at sea level in Yakutsk [6]), Moscow and in Europe did not exceed 20 % and attained about 40 % in Cheltenham. The latter point was located in the zone of maximal intensity increase [5]). During the increase of February 23, 1956 the greatest change was observed in Europe, the intensity of the hard component exceeding the usual value by 2—3 times. This means that this outburst was 5 to 10 times more intensive than that which occurred on November 19, 1949.

On basis of the experimental data obtained by us in the U.S.S.R. and by Dr T. C. Barton at Jamaica †, it can be concluded that the cosmic ray increase of February 23, 1956 was caused by charged particles, some of which certainly possessed energies exceeding 5 GeV and none of which apparently had energies exceeding $\approx$ 10—15 GeV.

At the boundary of the earth's atmosphere this additional flux of particles of the indicated energies should exceed the normal flux by approximately 100 times.

At present a detailed analysis of the experimental results is being carried out by the method of "coupling coefficients" [7]) of the primary and secondary variations of I. This should yield the energy spectrum of the additional cosmic ray flux at a sufficiently large distance from the earth, its time variation, the structure of the beam and the direction of the incoming particles. Data obtained at other stations recording the intensity of various cosmic ray components are important for this analysis which may yield information necessary for the elucidation of the mechanism of acceleration of charged particles to energies of the order of 10 GeV and also information concerning the mechanism of their emission from the sun and of their interaction with interplanetary and the terrestrial magnetic fields.

The authors express their deep gratitude to Professor S. N. Vernov for valuable advices and constant assistance rendered by him in the course of the present research.

† Private communication to Professor S. N. Vernov.

The authors take the opportunity to express on their own behalf as well as on that of Professor Vernov their appreciation to Dr Barton for communicating his data prior to publication.

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Note added in the proof.

As stated above, we were unable to establish a connection between the intensity increase of cosmic rays and chromosphere eruptions on the sun. According to the data received recently from the Tokyo Astronomical Observatory, a large chromosphere eruption (importance rating 3^+) was actually observed at the northwestern edge of the visible disk of the sun on February 23, 1956, starting at 3.31 Greenwich time.

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L/K-CAPTURE RATIO IN ^{153}Gd AND LOW-LYING LEVELS IN ^{153}Eu

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Abstract: The electron capture decay of ^{153}Gd has been investigated. Only one γ-ray, of energy 104 keV, was found to follow the electron capture decay; its K conversion coefficient resulted to be $\alpha_K = 0.70 \pm 0.03$. By using the integrating features of the scintillation spectrometer and by means of coincidence techniques the L- to K-capture ratio for the transition to the excited state of ^{153}Eu was obtained. It was found that $P_L/P_K = 0.68 \pm 0.02$ which corresponds to a transition energy $E_0 = 83 \pm 2$ keV. No branching to the ground state was found. A decay scheme is proposed. The disagreement between our results and those obtained by means of Coulomb excitation of ^{153}Eu is pointed out and briefly discussed.

1. Introduction

^{153}Gd is known [1]) to decay by orbital electron capture to ^{153}Eu with a half life of 236 d. The transition energy however is unknown

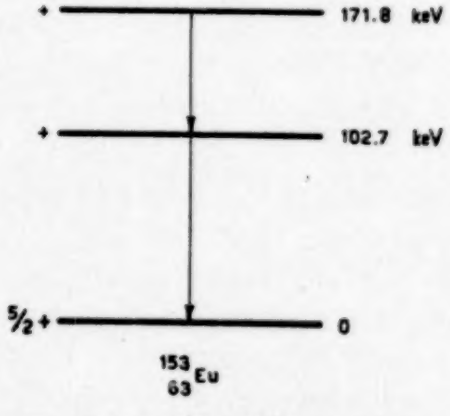

Fig. 1. Low-lying levels in ^{153}Eu from the decay of ^{153}Sm and ^{153}Gd.

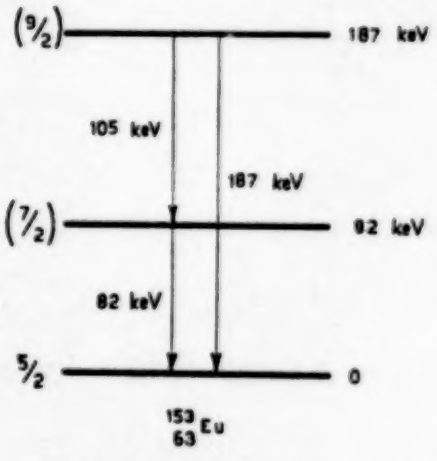

Fig. 2. Low-lying levels in ^{153}Eu from Coulomb excitation.

to date. The existence of a γ-ray transition of 104 keV in ^{153}Eu following the decay of ^{153}Gd was well established by several authors [1,2,3]). Church and Goldhaber [4]) have reported the presence

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of three γ-rays of energy 69.4, 97.3 and 103.1 keV. In addition it is well ascertained that the decay of ^{153}Sm is followed by γ-ray transitions of 70 and 103 keV energy [5, 6]). The above information suggests that a low-lying level scheme of ^{153}Eu, as shown in fig. 1, is likely to be correct.

The Coulomb excitation of Eu was carried out by Heydenburg and Temmer [7]) with α particles and by Mark and Paulissen [8]) with protons. Their results and conclusions substantially agree: γ-rays of energies 82, 105 and 187 keV (according to Heydenburg and Temmer) were observed and tentatively assigned to ^{153}Eu. The level scheme given by Heydenburg and Temmer is shown in fig. 2. This seems to represent a well developed rotational spectrum; actually ^{153}Eu has a strong nuclear deformation, as shown by its very high quadrupole moment and therefore the predictions of the "strong coupling" approximation of the unified model of Bohr and Mottelson should be verified.

The lack of agreement between the two different sets of information is quite evident. In particular, in one case the 104 keV γ-ray arises from the de-excitation of the first level of ^{153}Eu, in the other it represents a transition between the second and the first level. In view of these facts and in order to compare the results which can be deduced from both methods of investigation, a re-investigation of γ- and X-radiations from ^{153}Gd and a measurement of the unknown transition energy seemed us to be useful.

2. Measuring Techniques

Active Gd, supplied by AERE (Harwell), was obtained by irradiating Gd_2O_3 with slow neutrons. After the short-lived activities of ^{159}Gd (18 h) and ^{161}Tb (6.9 d) had disappeared, the only remaining activity was due to ^{153}Gd.

K X- and γ-radiations from ^{153}Gd were investigated by means of a single crystal spectrometer. Moreover two γ spectrometers were used in a coincidence arrangement to detect K X$-\gamma$ and K X$-$K X coincidences. The resolving power of the coincidence circuit was in the whole set of measurements kept at the fixed value of 2.7 μsec and such a selection of pulse heights was made that only pulses belonging to a single photopeak could in every case be used in order to trigger the coincidence circuit.

L X-radiations from the Gd sample were detected by means of a A+CH_4 filled proportional counter which could also be put in

coincidence with a single crystal γ spectrometer when L X$-\gamma$ coincidences had to be investigated.

3. Radiations Emitted from ^{153}Gd

The spectrum of K X- and γ-rays emitted from ^{153}Gd, as obtained by means of a single crystal γ spectrometer, is shown in fig. 3.

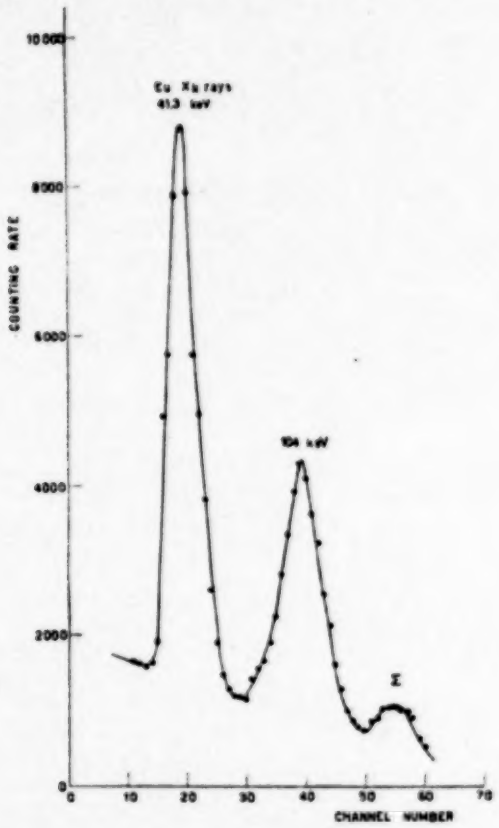

Fig. 3. γ-ray spectrum.

After calibration of the spectrometer, the energy of the γ line, as an average of several measurements, was 104 keV. The energy of the peak quoted as Σ peak corresponds to the sum of the energies of the γ line and of the characteristic K X line of Eu and was consequently interpreted as due to the simultaneous detection within the crystal of a γ quantum and a K X-ray. This fact is a well known consequence of the integrating features of a NaI crystal spectrometer and could be used to deduce the ratio of the probabilities of L and K electron capture processes in ^{153}Gd, as will be discussed in the next subsection.

a) *Determination of P_L/P_K ratio by the summation method*

The single crystal γ spectrometer can act as a coincidence device where the emission of a γ and a K X quantum is simultaneous. If the amplifying chain is fast enough, the resolving power of such a coincidence detector depends on the decay time of the light pulses from NaI (Tl) and is consequently 1.5 μsec or thereabout.

If $I_\gamma = I_{104} + I_\Sigma$ is the total intensity of 104 keV γ-rays and I_Σ the intensity of the Σ peak, we obtain

$$\frac{I_\gamma}{I_\Sigma} = \frac{1}{\omega_K \varepsilon_K}\left(1 + \frac{P_L}{P_K}\right), \tag{1}$$

where ω_K is the fluorescence yield of K shell in Eu ($\omega_K = 0.91$), ε_K the efficiency of the crystal spectrometer (cubic, 1″ size) and P_L and P_K the probabilities of L and K electron capture in the transition $^{153}Gd \rightarrow {}^{153}Eu$.

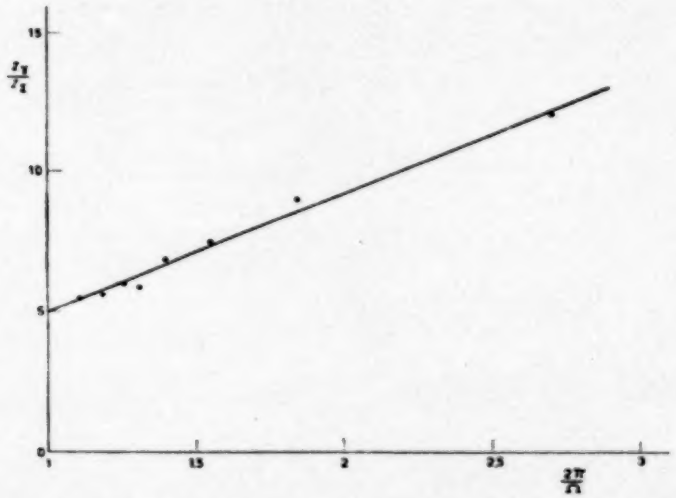

Fig. 4. Determination of the I_γ/I_Σ ratio by the summation method.

In fig. 4 the experimental values of the ratio I_γ/I_Σ are plotted versus the solid angle subtended by the detector: the extrapolation of source-detector distance to zero allows a safe evaluation of the efficiency ε_K which can consequently be assumed very close to 0.5. After correction for the probability of the I characteristic K X-rays escaping from the crystal the P_L/P_K ratio resulted to be 0.66 ± 0.03. This result will be used in deducing the energy of the electron capture transition.

b) *Determination of the ratio between the intensities of γ-rays and K X quanta*

Another datum which can be deduced from the inspection of the spectrum emitted in the decay of ^{153}Gd is the I_K/I_γ ratio, where I_K is the intensity of K X-rays and I_γ is the intensity of γ-rays:

$$I_K/I_\gamma = 1.65 \pm 0.02.$$

This result will be considered together with the data which were obtained from the coincidence measurements.

4. Coincidence Measurements with Crystal Spectrometers

Both the γ-ray peak and the peak due to K X radiations were used to trigger the coincidence circuit. In this section the formulae which were used during the elaboration of experimental data and the numerical results will be given.

It is assumed a priori that besides the electron capture transition towards the 104 keV excited state of ^{153}Eu, a direct electron capture transition to the ground state of ^{153}Eu is also possible; let N' and N'' be the respective numbers of transitions which take place within the active sample. If $I_{K\gamma}$ and I_{KK} are the counting rates due to K X$-\gamma$ and K X$-$KX coincidences, α_K and α the K and the total internal conversion coefficients for the γ transition, $(P_L/P_K)'$ and $(P_L/P_K)''$ the P_L/P_K ratios for the two possible electron captures, we obtain

$$I_\gamma = N' \varepsilon_\gamma \frac{1}{1+\alpha} \tag{2}$$

$$I_K = N' \omega_K \varepsilon_K \left[\frac{1}{1+\left(\frac{P_L}{P_K}\right)'} + \frac{\alpha_K}{1+\alpha} + \frac{N''}{N'} \frac{1}{1+\left(\frac{P_L}{P_K}\right)''} \right] \tag{3}$$

$$I_{K\gamma} = N' \varepsilon_\gamma \varepsilon_K \omega_K \frac{1}{1+\alpha} \frac{1}{1+\left(\frac{P_L}{P_K}\right)'} \tag{4}$$

$$I_{KK} = 2N' \omega_K^2 \varepsilon_K \bar{\varepsilon}_K \frac{\alpha_K}{1+\alpha} \frac{1}{1+\left(\frac{P_L}{P_K}\right)'} \tag{5}$$

whence we have

$$\alpha_K = \frac{1}{2}\frac{\varepsilon_\gamma}{\omega_K \bar{\varepsilon}_K}\frac{I_{KK}}{I_{K\gamma}} \tag{6}$$

$$\left(\frac{P_L}{P_K}\right)' = \varepsilon_K \omega_K \frac{I_\gamma}{I_{K\gamma}} - 1 \tag{7}$$

$$\frac{I_K}{I_\gamma} = \frac{\omega_K \varepsilon_K}{\varepsilon_\gamma}(1+\alpha)\left[\frac{1}{1+\left(\frac{P_L}{P_K}\right)'} + \frac{\alpha_K}{1+\alpha} + \frac{N''}{N'}\frac{1}{1+\left(\frac{P_L}{P_K}\right)''}\right]. \tag{8}$$

It will be noticed that (7) is equivalent to (1) and should consequently give a consistent result.

After correction for the I K X-rays escaping from the crystal we find

$$I_{KK}/I_{K\gamma} = 1.28 \pm 0.06,$$
$$I_\gamma/I_{K\gamma} = 4.14 \pm 0.09,$$

which in turn give

$$\alpha_K = 0.70 \pm 0.03$$
$$(P_L/P_K)' = 0.70 \pm 0.03.$$

The value of α_K obtained here agrees with the values given by Siegbahn [9]), Lee and Katz [6]), but considerably differs from that found by Hein and Voigt [1]), Sunyar [5]) and McGowan [5]). The multipole character of the 104 keV transition cannot be uniquely assigned from the above quoted value of α_K though only an M1 or E2 or M1+E2 transition is consistent with it.

In order to deduce any information about the branching ratio N''/N', a value for the total conversion coefficient α has to be assumed. A K/L conversion ratio equal to 5.3 was taken according to the results of Siegbahn [9]), Mihelic [10]), Rutledge, Cork and Burson [11]), Cork, LeBlanc, Nester and Stumpf [2]). Such a value of K/L, if compared with our result for α_K, gives $\alpha_L = 0.13$ and $\alpha = 0.83$. Moreover it is to be noticed that from $(P_L/P_K)' = 0.68 \pm 0.02$, as it results from the mean value of the two independent determinations, the energy involved in the electron capture transition from the ground state of ^{153}Gd to the 104 keV excited state of ^{153} Eu, can be calculated according to the formula given by Marshak [12])

$$\frac{P_L}{P_K} = \left(\frac{g_{L_I}}{g_K}\right)^2 \left(\frac{E_0 - E_L}{E_0 - E_K}\right)^2 \left(1 + \frac{f^2_{L_{II}}}{g^2_{L_I}}\right). \tag{9}$$

From (9) and from

$$\left(\frac{g_{L_I}}{g_K}\right)^2 = 0.128 \text{ [13]}$$

$$\left(\frac{f_{L_{II}}}{g_{L_I}}\right)^2 = \frac{3}{16}\left(\frac{Z-4.15}{137.3}\right)^2, \text{ [12]}$$

and with the known values of E_L and E_K, we obtain for the transition energy E_0

$$E_0 = 83 \pm 2 \text{ keV}.$$

It can be consequently assumed that the total energy available for the decay of ^{153}Gd is 187 ± 3 keV, and hence a $(P_L/P_K)''$ ratio equal to 0.23 can be calculated from (9).

If the above-indicated values for α and $(P_L/P_K)''$ are inserted into (8), and I_K/I_γ is taken to be 1.65 ± 0.02, according to the result reported in section 3, a branching ratio N''/N' very close to zero can be deduced. A similar and quite consistent conclusion is drawn from the comparison of (3) and (4); consequently it seems reasonable to state that the electron capture transition $^{153}\text{Gd} \rightarrow {}^{153}\text{Eu}$ takes place with a branching ratio of less than 3 percent, if any.

5. L X-γ Coincidences

L X radiations, which are emitted both in L capture transition to the excited state of ^{153}Eu and in the L conversion process of the 104 keV γ line, could be easily detected with a A+CH_4 filled proportional counter, and coincidence measurements were possible between the L X line and the γ line. The obtained results can be used to deduce a mean L-fluorescence yield $\bar{\omega}_L$ of Eu, according to the method developed by Robinson and Fink [14].

$\bar{\omega}_L$ is given by

$$I_{L\gamma}/I_\gamma = (P_K n_{LK} + P_L)\bar{\omega}_L \varepsilon_L$$

where $I_{L\gamma}$ is the intensity of L X$-\gamma$ coincidences, ε_L the efficiency of L X-rays detector and n_{LK} the number of L-shell vacancies produced in filling a K-shell vacancy. With the experimental value $I_{L\gamma}/I_\gamma = 4.37 \times 10^{-2}$ and considering $n_{LK} = 0.85$ and $\varepsilon_L = 0.28$ we get $\bar{\omega}_L = 0.17$.

The obtained value of $\bar{\omega}_L$ is in satisfactory agreement with the data recently discussed by Robinson and Fink [14].

6. Decay Scheme and Discussion

Spin and parity of the ground level of ^{153}Eu are known to be 5/2+. As regards the ground state of ^{153}Gd, no unambiguous assignement of the orbital can be made on the basis of the nuclear shell model owing to the fact that the two available levels, $f_{7/2}$ and $h_{9/2}$, lie very close to each other. However a 9/2− orbital should be preferable as no appreciable branching between the ground states of ^{153}Gd and ^{153}Eu was observed.

A serious difficulty to this interpretation arises however from the fact that taking the energy of the electron capture process to be 83 ± 2 keV and from graphes, and normographs given by Moszkowski [15]) the log *ft* of the electron capture transition to the excited level of ^{153}Eu is

$$\log ft = 5.4$$

and this is what is expected for an allowed transition ($\Delta I = 0, 1$; no).

From the multipolarity of the γ transition (E2 or M1 or E2+M1) the parity of the excited state of ^{153}Eu is uniquely determined to be the same as of the ground state of ^{153}Eu and this conflicts with the fact that no change of parity should take place in the electron capture transition.

According to the classification discussed by Mottelson and Nilsson [16]), high values of the spin of ground state of ^{153}Gd are strongly disfavoured. The orbitals 1/2+ and 3/2− seem to be most probable for it and the condition that no parity change should occur in the electron capture transition favours the choice of the 1/2+ orbital. Correspondingly the orbital for the excited level of ^{153}Eu would seem to be 1/2, 3/2+, and such an assignation does not conflict with the known spin and parity of ground state of ^{153}Eu and with the multipole character of the γ transition.

Also the non-existence of an electron capture transition between the ground states of ^{153}Gd and ^{153}Eu can be easily explained in this way, because a transition ($\Delta I = 2$, no) is expected to be highly forbidden.

The decay scheme given in fig. 5 is consequently proposed.

Anyway, the proposed decay scheme implies an excited level of ^{153}Eu at 104 keV in strict agreement with previous results of β- and γ-ray spectrometry. As regards the 173 keV level which has been put in evidence in the decay of ^{153}Sm, its observation in the

decay of ^{153}Gd is prevented in view of the obtained value for the atomic mass difference between ^{153}Gd and ^{153}Eu. Actually a K-capture to this level is energetically impossible and a pure L-capture, even if energetically possible, should occur with very low probability.

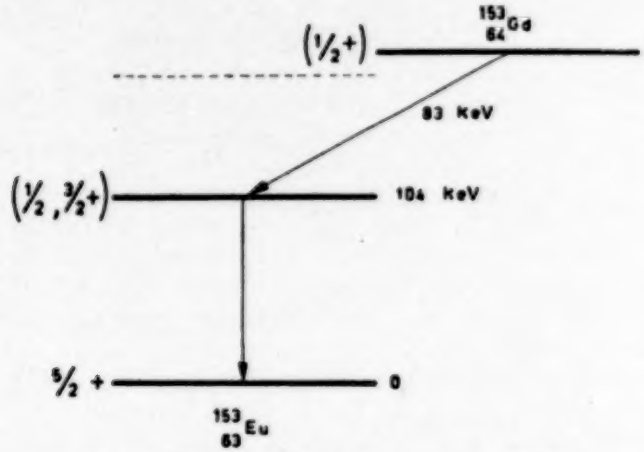

Fig. 5. Proposed decay scheme of ^{153}Gd.

Our value for the atomic mass difference, if correct, emphasizes once more the disagreement between the results obtained by means of β- and γ-ray spectrometry and those obtained through Coulomb excitation. It cannot be considered to date as definitely sure that the same level pattern must be excited both in the Coulomb excitation of a nucleus and throug EC decay from the neighbouring nucleus and consequently the lack of agreement may not need explanation. Anyway some remarks about the disagreement can be made. The assignement of the rotational band of fig. 2 to ^{153}Eu, and not to ^{151}Eu, is not completely certain because, as outlined by Mark and Paulissen [8]), no enriched isotopes of Eu were available for those experiments. Furthermore if in the level scheme of fig. 2 the 82 and 104 keV γ-rays were interchanged a reasonable agreement with the present measurements would be achieved, but the rotational family would seem to be destroyed.

An attempt to build up a rotational family with the 104 and 173 keV levels can be made by considering that the ratio between the energies of these levels is the same as the ratio of third-to-second excited state energy according to the predictions of the unified model. Obviously this would require a first excited state at $\approx$ 46 keV, but we did not find any experimental evidence for postulating such a level.

We would like to thank prof. G. Bolla for his kind interest in the present work.

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COURBES D'EXCITATION DE LA RÉACTION $^{16}O(d, p)^{17}O^*$

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Abstract: Excitation curves of the reaction $^{16}O(d, p)$ $^{17}O^*$ (875 keV level) are given at angles 10° and 135° (in the laboratory system) for deuteron energies between 1.7 and 4.5 MeV.

1. Introduction

L'étude des réactions (d, p) conduit à des résultats en apparence contradictoires: d'une part, la section efficace d'une telle réaction passe par un maximum (résonance) pour chaque énergie de deutérons correspondant à certains niveaux du noyau composé (noyau cible+deutéron); d'autre part, la forme de la distribution angulaire de la même réaction s'interprète très bien dans le cadre de la théorie de Butler [1]) fondée sur l'hypothèse du "stripping" sans formation du noyau composé.

Pour mieux comprendre le mécanisme de ces réactions (d, p) nous avions, en 1953, en collaboration avec d'autres auteurs, étudié les distributions angulaires de la réaction

$$^{16}O(d, p)\ ^{17}O^*, \text{ niveau à } 875 \text{ keV} \qquad (1)$$

pour des énergies de deutérons voisines de l'énergie d'une résonance de ^{18}F (2.06 MeV) et nous avions pu montrer que ces distributions angulaires étaient en accord avec la théorie de Butler même à l'énergie de résonance [2]).

Ces résultats s'interprètent bien dans le cadre de théories plus complexes que celles de Butler [3, 4, 5]). En effet, elles conduisent à des distributions angulaires assez peu différentes de celles de la théorie de Butler compte tenu de l'indétermination qui subsiste sur la valeur du rayon nucléaire. Par contre, elles prennent en considération les interactions entre proton et noyau final, deutéron et noyau initial, ce qui permet une évaluation plus exacte des valeurs absolues des sections efficaces.

La théorie d'Horowitz et Messiah [3]) fut la première à tenir compte de telles interactions; les calculs étaient conduits par ces auteurs en considérant l'interaction proton-noyau comme une interaction de sphère dure; dans le cas d'une interaction résonante, la courbe de variation de la section efficace (d, p) présentera un accident au voisinage de l'énergie correspondant au niveau du noyau composé.

Tobocman [4]) a pu tenir compte des interactions deutéron-noyau initial et proton-noyau final ainsi que des effets coulombiens. Ses formules permettent de calculer la distribution angulaire au sommet d'une résonance et les valeurs absolues de la section efficace différentielle $d\sigma/d\Omega$, connaissant le moment orbital l du neutron capturé correspondant à cette résonance: c'est ce qui a été effectivement réalisé par Tobocman et Kalos [6]) dans le cas de la réaction $^{19}F(d, p)\,^{20}F$, en utilisant une machine à calculer électronique.

Thomas [5]) calcule les sections efficaces différentielles des réactions (d, p) au voisinage des résonances et ses formules devraient permettre d'obtenir la variation de ces sections efficaces à divers angles en fonction de l'énergie; mais aucun calcul numérique n'a encore été effectué à notre connaissance.

Comme nous l'avions annoncé antérieurement, nous avons repris l'étude de la réaction (1) en nous attachant à comparer les courbes d'excitation représentant les variations des sections efficaces différentielles $d\sigma/d\Omega$ à 10° et 135° (système du laboratoire) en fonction de l'énergie des deutérons. Nous espérions ainsi obtenir des résultats permettant une confrontation plus quantitative avec les théories du stripping en précisant par ailleurs les niveaux de ^{18}F dans cette région.

2. Dispositif expérimental

Il est schématisé fig. 1. Les deutérons sont accélérés par le Van de Graaff de Saclay. Les fentes d'entrée et de sortie de l'aimant analyseur du Van de Graaff sont réglées pour donner une résolution en énergie de l'ordre de $1\,^0/_{00}$. L'énergie des deutérons est mesurée par l'intermédiaire de la fréquence de résonance magnétique nucléaire, proportionnelle au champ magnétique de déflection. La relation énergie-fréquence est déterminée par étalonnage au moyen du seuil de la réaction $^7Li(p, n)\,^7Be$.

La chambre à cible, isolée du Van de Graaff et des appareils détecteurs, forme cage de Faraday. Elle est reliée à un intégrateur

de charges qui permet la mesure du courant d'ions arrivant sur la cible.

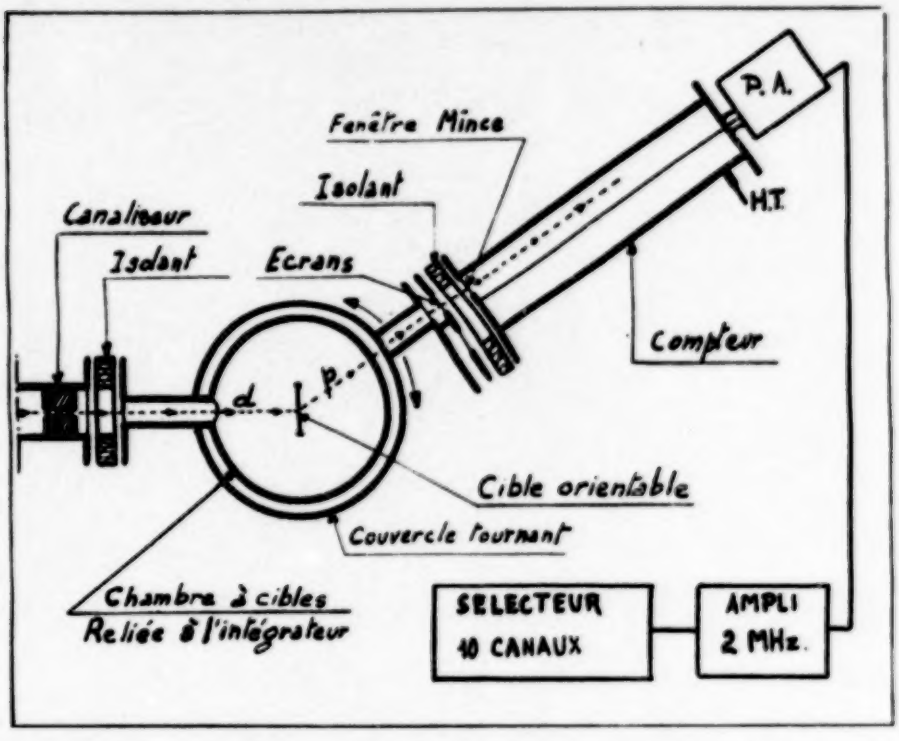

Fig. 1. Le dispositif expérimental.

Un courant de 2 μA ou de 0.1 μA suivant les cibles est utilisé pour ces expériences. Les cibles sont de l'oxyde de baryum: le baryum est déposé par évaporation thermique sur un support; il est ensuite oxydé dans une atmosphère d'oxygène purifié. Le support, variable avec l'angle d'observation des protons de la réaction, est, soit du tantale épais (observation vers l'arrière), soit une feuille d'or mince ($\approx 500\,\mu g/cm^2$) (observation vers l'avant). Le dépôt de BaO a une épaisseur de quelques keV pour des deutérons de 3 MeV. Les protons sont détectés par un compteur proportionnel fermé par une fenêtre mince d'aluminium. Un jeu d'écrans d'aluminium placés sur le trajet des protons permet d'arrêter les deutérons diffusés par la cible et de ralentir suffisamment les protons pour que leur parcours ne dépasse pas la longueur du compteur. Une source escamotable de particules α du plutonium, placée dans le compteur, permet de contrôler et d'étalonner ce compteur. Un exemple de spectre obtenu avec les protons de la réaction est reproduit (fig. 2).

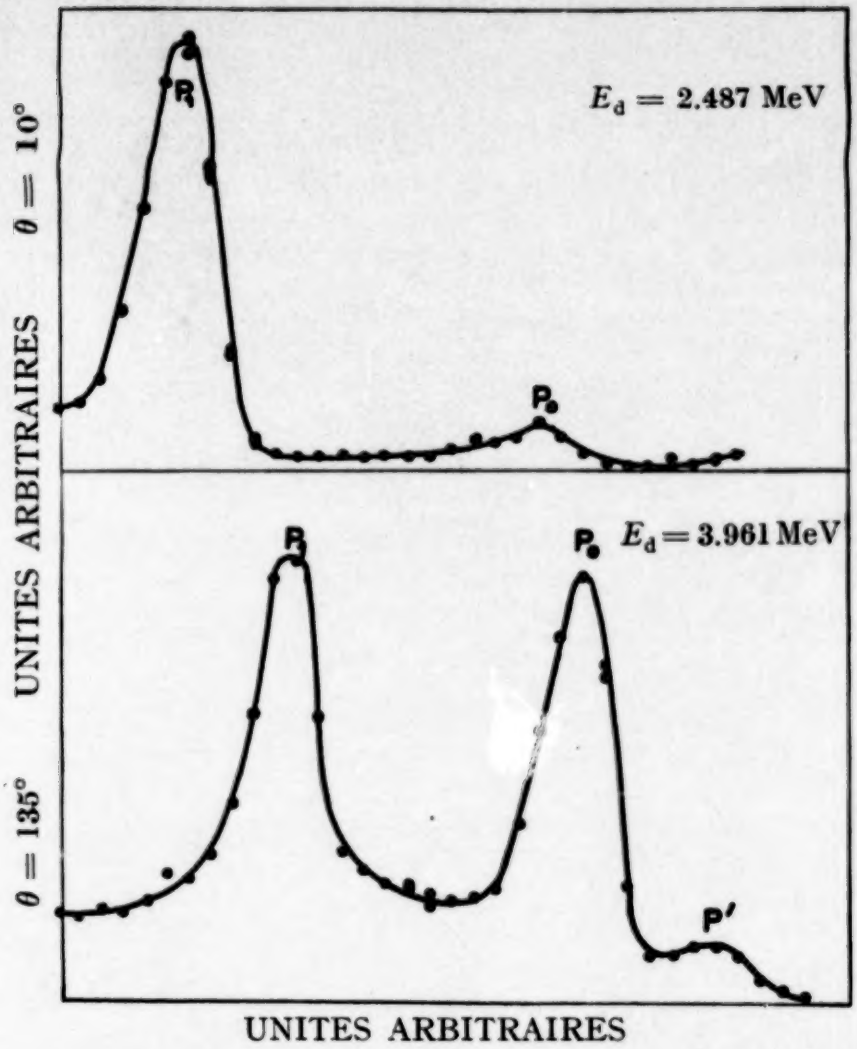

Fig. 2. Spectres de protons

P_0: protons de $^{16}O(d, p)\ ^{17}O$ (fondamental)

P_1: protons de $^{16}O(d, p)\ ^{17}O^*$ (niveau de 875 keV)

P': protons de $^{12}C(d, p)\ ^{13}C$ (fondamental)

3. Résultats

La figure 3 reproduit les courbes d'excitation à 10° et 135°. En ordonnées on porte, en unités arbitraires, le nombre de protons émis à l'angle considéré, c'est-à-dire la surface du pic (fig. 2) correspondant à la réaction pour une énergie donnée E_d de deutérons incidents.

La barre d'erreur sur chaque point correspond:

(*a*) aux erreurs statistiques dues au comptage (de l'ordre de 1.5 %);

(*b*) aux erreurs sur la mesure relative du courant de deutérons incidents (estimées inférieures à 1 % d'après la mesure des courants de fuite);

(*c*) aux variations du nombre de noyaux cibles correspondant à des déplacements du faisceau sur une cible non homogène (estimées à environ 7 % d'après les différences d'ordonnées obtenues dans les expériences faites à une même énergie);

(*d*) aux erreurs sur l'estimation du fond soustrait pour calculer la surface du pic.

L'erreur résultante peut atteindre 10 %.

L'examen des deux courbes fait apparaître un grand nombre de résonances dans le domaine d'énergie étudié. Aucune résonance ne se détache bien du fond. Celle à 2.05 MeV, qui peut sembler unique sur la courbe à 135°, apparaît multiple après l'étude faite à 10°.

TABLE I

Stratton[9]		Bonner[10]	Le présent travail		$^{18}F^*$
$^{16}O(d, p)^{17}O$	$^{16}O(d, p)^{17}O^*$ 875 keV	$^{16}O(d, n)^{17}F$	$^{16}O(d, p)^{17}O^*$ (875 keV)		
à 53° E_d (MeV)	à 53° E_d (MeV)	à 10° E_d (MeV)	à 10° E_d (MeV)	à 135° E_d (MeV)	(MeV)
			1.86		9.18
			2.045	2.051	9.35
			2.14		9.43
				2.34	
2.5			2.56	2.55	9.80
	2.61				
			2.72	2.78	9.95
2.80	2.87				
	2.94		2.93		10.13
3.0		3.1	3.16	3.11	10.34
		3.23			
3.3			3.36	3.32	10.52
	3.39				
3.45		3.4		3.43	
	3.69	3.62	3.61	3.69	10.74
			3.75		10.86
				3.83	
		3.94	3.95	3.93	11.04
				4.00	
		4.1	4.1		11.17
			4.35		11.4

Le tableau des résultats ci-dessus donne les énergies des maxima correspondant aux résonances, ainsi que l'énergie d'excitation

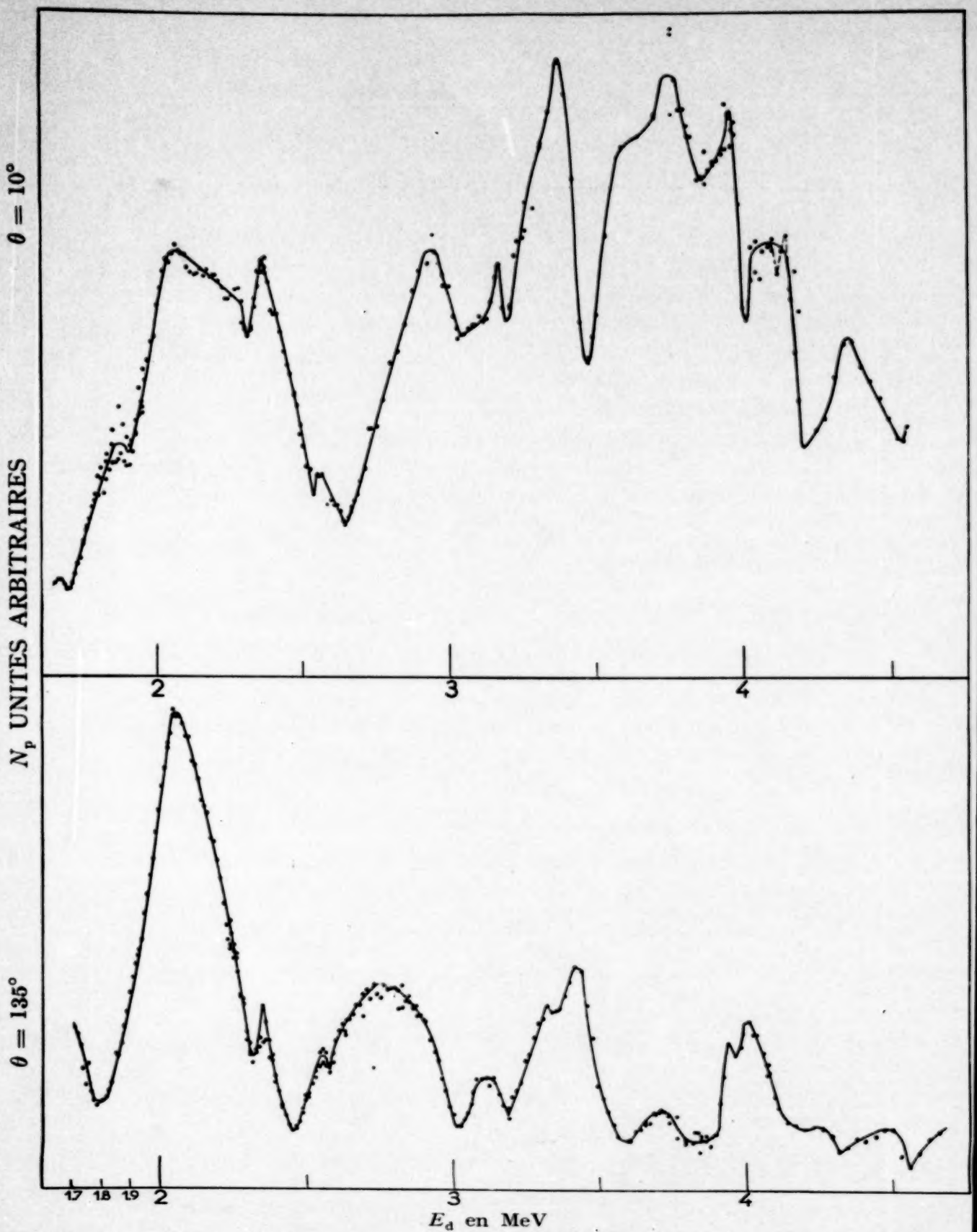

Fig. 3. Courbes d'excitation.

correspondante pour le noyau ^{18}F et permet une comparaison avec différents auteurs. On remarque que les énergies des résonances ne sont pas les mêmes à 10° et 135° et ne correspondent pas toutes aux résultats des autres auteurs.

Un mécanisme d'interférence peut expliquer de telles différences. Nos expériences ne donnent donc que des indications sur l'énergie des niveaux de ^{18}F.

Pour obtenir avec précision ces niveaux, il faudrait faire des calculs analogues à ceux utilisés par exemple dans le cas de diffusions élastiques (p, p); on devrait obtenir ainsi les énergies exactes des niveaux du noyau composé, leurs largeurs, leurs spins et parités. Les difficultés d'une telle étude tiennent à la complexité des calculs.

Il serait intéressant (comme Thomas entre autres, l'a suggéré) d'étudier parallèlement les diverses réactions et diffusions élastiques conduisant aux mêmes niveaux du noyau composé, par exemple pour ^{18}F: $^{14}N(\alpha, \alpha)$, $^{17}O(p, p)$, $^{16}O(d, d)$, $^{16}O(d, p)$, $^{17}O(p, d)$.

Il semble raisonnable de se livrer à l'analyse théorique dans un cas expérimental où certains niveaux du noyau composé sont déjà connus ainsi que les spins et parités. C'est le cas de la réaction $^{12}C(d, p)^{13}C$ qui a été étudiée avec soin[7, 8]). Cette analyse a été tentée [8]) pour une résonance de ^{14}N au voisinage de laquelle des courbes d'excitation avaient été tracées pour 12 angles différents, en utilisant une formule approchée séparant l'amplitude résonante de l'amplitude non résonante.

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THE WAVE FUNCTIONS IN COULOMB FIELDS

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Abstract: In this paper a detailed mathematical study of the confluent hypergeometric functions is carried out. Exact analytic expansions in series of Bessel functions are developed. These expansions are found to be useful for the computation of the wave functions in Coulomb fields. They have the advantage over power series expansions in the clearance of imaginaries from the wave functions for repulsive Coulomb fields.

1. Introduction

The radial equation for an electron of energy E and mass m moving in the Coulomb field of a nucleus of charge Ze is

$$\frac{\mathrm{d}^2W}{\mathrm{d}r^2}+\left\{\frac{2m}{\hbar^2}\left(E+\frac{Ze^2}{r}\right)-\frac{l(l+1)}{r^2}\right\}W=0, \tag{1}$$

where W is r times the radial factor of the wave function and $l\hbar$ is the angular momentum of state considered. By the substitutions

$$E=-\frac{me^4Z^2}{2\hbar^2k^2}, \quad r=\frac{\hbar^2k}{2me^2Z}\,z \text{ and } l=m-\tfrac{1}{2} \tag{2}$$

this equation transforms into

$$\frac{\mathrm{d}^2W}{\mathrm{d}z^2}+\left\{-\frac{1}{4}+\frac{k}{z}+\frac{\frac{1}{4}-m^2}{z^2}\right\}W=0 \tag{3}$$

which is the confluent hypergeometric equation discussed by Whittaker and Watson [1]). In their notation, two solutions of this equation near $z=0$ are $M_{k,m}(z)$ and $M_{k,-m}(z)$. If Coulomb's law applies for all values of r, then only the regular solution $M_{k,m}(z)$ is needed. On the other hand, when this law is modified for small values of r both solutions are required. Since $2m$ is an integer, as inferred from eq. (2), $M_{k,-m}(z)$ is infinite. For this reason, a second particular solution of eq. (3) will be defined.

In problems requiring numerical estimates, the radial wave functions are usually approximated to Bessel functions. This approximation is based on the fact that the zero energy radial equation

is a Bessel equation. Yost, Wheeler and Breit [2]) tried to extend the range of approximation to cover small energies by developing an expansion of the function $M_{k,\,m}$ into a series of Bessel functions, in which the coefficients are powers of $1/k^2$ i.e. powers of the energy. They obtained, after laborious work, a few terms in that expansion and the terms were found to grow cumbrously. In the present work, exact analytic expansions of the solutions of eq. (3) in series of Bessel functions are obtained. The attempt is based on a relation given by Whittaker and Watson between Bessel functions and the function $M_{0,\,m}(z)$, in the special case $k = 0$, and expressed as follows:

$$M_{0,\,m}(z) = 2^{2m}\,\Gamma(m+1)z^{\frac{1}{2}}i^{-m}J_m(\tfrac{1}{2}iz). \tag{4}$$

2. A Second Particular Solution of the Confluent Hypergeometric Equation

Let $I_{k,\,m}(z)$ denote the function

$$I_{k,\,m}(z) = \frac{1}{\Gamma(1+2m)}M_{k,\,m}(z). \tag{5}$$

The two functions $I_{k,\,m}(z)$ and $I_{k,\,-m}(z)$ obviously satisfy eq. (3). When $2m$ is an integer they are a constant multiple of each other i.e.

$$I_{k,\,-m}(z) = (-)^{2m}\frac{\Gamma(\frac{1}{2}+m+k)}{\Gamma(\frac{1}{2}-m+k)}I_{k,\,m}(z).$$

In order to have two distinct solutions in this case, a function $N_{k,\,m}(z)$ defined by the equation

$$N_{k,\,m}(z) = \frac{\dfrac{\Gamma(\frac{1}{2}+m+k)}{\Gamma(\frac{1}{2}-m+k)}I_{k,\,m}(z)\cos 2m\pi - I_{k,\,-m}(z)}{\sin 2m\pi} \tag{6}$$

is introduced. This function is a solution of eq. (3) when $2m$ is not an integer. When $2m$ is an integer, it is defined by the limiting form of this equation, viz.

$$N_{k,\,m}(z) = \frac{1}{2\pi}\frac{\partial}{\partial m}\left\{\frac{\Gamma(\frac{1}{2}+m+k)}{\Gamma(\frac{1}{2}-m+k)}I_{k,\,m}(z)\right\} - (-)^{2m}\frac{1}{2\pi}\frac{\partial}{\partial m}\{I_{k,\,-m}(z)\}, \tag{7}$$

where $2m$ is made an integer after differentiation. On developing the ascending series in z for this function, the following expansion is obtained:

$$\pi N_{k,m}(z) = -e^{-\frac{1}{2}z} z^{\frac{1}{2}-m} \sum_{n=1}^{2m} \frac{\Gamma(\frac{1}{2}+m+k)\Gamma(2m-n+1)}{\Gamma(n)\Gamma(\frac{3}{2}+m+k-n)} z^{n-1}$$
$$+e^{-\frac{1}{2}z} z^{\frac{1}{2}+m} \sum_{n=1}^{\infty} \frac{(-)^{n-1}\Gamma(\frac{1}{2}+m+k)z^{n-1}}{\Gamma(n)\Gamma(2m+n)\Gamma(\frac{3}{2}-m+k-n)} \quad (8)$$
$$\times\{\log z - \psi(n) - \psi(2m+n) + \psi(\tfrac{3}{2}-m+k-n)\},$$

where ψ is the logarithmic derivative of the gamma function.

The function $I_{k,m}(z)$ and $N_{k,m}(z)$ are fundamental solutions of eq. (3) when $2m$ is an integer.

3. An Expansion of $I_{k,m}(z)$ in a Series of Bessel Functions

By suitably altering the contour integral for Whittaker's function $W_{k,m}(z)$, the function $I_{k,m}(z)$ can be represented by the contour integral

$$I_{k,m}(z) = -\frac{1}{2\pi i} e^{-\frac{1}{2}z} z^{\frac{1}{2}+m} \int_{\infty}^{(0+)} (-t)^{-2m-1} \left(1+\frac{z}{t}\right)^{-\frac{1}{2}-m+k} e^{-t}\,dt, \quad (9)$$

where the path of integration starts at infinity on the real axis, encircles the origin in the positive direction and returns to plus infinity; on this path $-\pi \leqq \arg(-t) \leqq \pi$ and $|z| < |t|$. The factor $(1+z/t)^{-\frac{1}{2}-m+k}$ in the integrand can then be expanded in powers of z/t and term-by-term integration can be performed, thus giving the power series for $I_{k,m}(z)$.

Substituting $k = 0$ in (9),

$$I_{0,m}(z) = -\frac{1}{2\pi i} e^{-\frac{1}{2}z} z^{\frac{1}{2}+m} \int_{\infty}^{(0+)} (-t)^{-2m-1} \left(1+\frac{z}{t}\right)^{-\frac{1}{2}-m} e^{-t}\,dt. \quad (10)$$

An attempt is now made to expand the integral for $I_{k,m}(z)$ in a series of $I_{0,m}(z)$ integrals. The factor $(1+z/t)^k$ in the integrand of the integral (9) is expanded in a Maclaurin's series of $(z/t)(1+z/t)^{-\frac{1}{2}}$, viz:

$$\left(1+\frac{z}{t}\right)^k = \sum_{r=0}^{\infty} \frac{k\Gamma\left(k+\frac{r}{2}\right)}{r!\Gamma\left(k-\frac{r}{2}+1\right)} \left(\frac{z}{t}\right)^r \left(1+\frac{z}{t}\right)^{-\frac{r}{2}}.$$

This power series expansion converges absolutely and uniformly along the contour, provided that

$$\left|\frac{z}{t}\left(1+\frac{z}{t}\right)^{-\frac{1}{2}}\right| < 2;$$

but since

$$\left|\frac{z}{t}\left(1+\frac{z}{t}\right)^{-\frac{1}{2}}\right| \leqq \left|\frac{z}{t}\right|\left(1-\left|\frac{z}{t}\right|\right)^{-\frac{1}{2}}$$

it follows that

$$\left|\frac{z}{t}\left(1+\frac{z}{t}\right)^{-\frac{1}{2}}\right| < 2 \text{ if } \left|\frac{z}{t}\right| < 2\sqrt{2}-2 < 1.$$

Thus on the contour of integration the condition that $|z| < |t|$ is preserved. Further, along the contour $|e^{-t}| \leqq 1$ for $R(t) \geqq 0$ and $|e^{-t}| \leqq N$ for $R(t) \leqq 0$, where N is independent of t, so that by Weierstrass's test for uniform convergence the integrand of the integral for $I_{k,m}(z)$ has been expanded in a uniformly convergent series with regard to t, and so term-by-term integration is permissible, thus:

$$I_{k,m}(z) = -\frac{1}{2\pi i} e^{-\frac{1}{2}z} z^{\frac{1}{2}+m} \int_{\infty}^{(0+)} \sum_{r=0}^{\infty} (-)^r \frac{k\Gamma\left(k+\frac{r}{2}\right) z^r}{r!\Gamma\left(k-\frac{r}{2}+1\right)} (-t)^{-1-2m-r} \left(1+\frac{z}{t}\right)^{-\frac{1}{2}-m-\frac{r}{2}} e^{-t} dt$$

$$= -\frac{1}{2\pi i} e^{-\frac{1}{2}z} z^{\frac{1}{2}+m} \sum_{r=0}^{\infty} (-)^r \frac{k\Gamma\left(k+\frac{r}{2}\right) z^r}{r!\Gamma\left(k-\frac{r}{2}+1\right)} \int_{\infty}^{(0+)} (-t)^{-1-2m-r} \left(1+\frac{z}{t}\right)^{-\frac{1}{2}-m-\frac{r}{2}} e^{-t} dt$$

$$= \sum_{r=0}^{\infty} (-)^r \frac{k\Gamma\left(k+\frac{r}{2}\right)}{r!\Gamma\left(k-\frac{r}{2}+1\right)} z^{\frac{r}{2}} I_{0,m+\frac{r}{2}}(z),$$

by (10). But, from eqs. (4) and (5) and Legendre's duplication formula for the Gamma-function

$$I_{0,m}(z) = \frac{(\pi z)^{\frac{1}{2}}}{\Gamma(\frac{1}{2}+m)} i^{-m} J_m(\tfrac{1}{2}iz)$$

$$= \frac{(\pi z)^{\frac{1}{2}}}{\Gamma(\frac{1}{2}+m)} I_m(\tfrac{1}{2}z),$$

where I_m is the modified Bessel function of the first kind, and so

$$I_{k,m}(z) = (\pi z)^{\frac{1}{2}} \sum_{r=0}^{\infty} \frac{(-)^r k\Gamma\left(k+\frac{r}{2}\right) z^{\frac{r}{2}}}{r!\Gamma\left(k-\frac{r}{2}+1\right)\Gamma\left(\frac{1}{2}+m+\frac{r}{2}\right)} I_{m+\frac{r}{2}}\left(\frac{z}{2}\right)$$

$$= (\pi z)^{\frac{1}{2}} \left\{ \frac{1}{\Gamma(\frac{1}{2}+m)} I_m(\tfrac{1}{2}z) - \frac{kz^{\frac{1}{2}}}{1!\Gamma(1+m)} I_{m+\frac{1}{2}}(\tfrac{1}{2}z) + \frac{k^2 z}{2!\Gamma(\frac{3}{2}+m)} I_{m+1}(\tfrac{1}{2}z) - \dots \right\}. \tag{11}$$

This expansion of $I_{k,\,m}(z)$ in a series of Bessel functions is of quite a significant value. It is valid for all values of m and k. As seen, the convergence of the series is quite rapid since the I_m functions decrease as their order increase and, moreover, each term in the expansion contains two factorial products in its denominator. For large values of k, the convergence may appear slow, but this increase in k is counterbalanced by a decrease in z as inferred from eq. (2).

4. An Expansion of $N_{k,\,m}(z)$ in Series of $I_{k,\,m}(z)$ Functions

An expansion of $N_{k,\,m}(z)$ in series of Bessel functions may be obtained by substituting in eq. (6) the series in Bessel functions for $I_{k,\,m}(z)$ and $L_{k,\,-m}(z)$ as given in eq. (11). But it seems difficult to express the limiting form of $N_{k,\,m}(z)$ when $2m$ is an integer, in series of Bessel functions. The reason for this difficulty arises from the fact that, in case of $2m$ being an integer, the series (11) does not show that $I_{k,\,m}(z)$ is a constant multiple of $I_{k,\,-m}(z)$. However, one can express the limiting form of $N_{k,\,m}(z)$ in series of $I_{k,\,m}(z)$ functions; the latter functions have been already expressed in terms of Bessel functions.

For convenience $I_{k,\,m}(z)$ is expanded as follows:

$$I_{k,\,m}(z) = z^m \{a_0 I_{k,\,0}(z) + a_{\frac{1}{2}} z^{\frac{1}{2}} I_{k,\,\frac{1}{2}}(z) + a_1 z I_{k,\,1}(z) + \dots\},$$

where $a_0, a_{\frac{1}{2}}, a_1, \dots$ are independent of z. On equating coefficients of $z^{\frac{1}{2}+m}, z^{\frac{3}{2}+m}, z^{\frac{5}{2}+m}, \dots$ on each side of this equation, the factors $a_0, a_{\frac{1}{2}}, a_1, \dots$ are determined uniquely in turn. They are simply given by

$$a_{\frac{r}{2}} = \frac{1}{r!(2m+r)\Gamma(2m)} \frac{\Gamma\left(k+\frac{r}{2}+\frac{1}{2}\right)}{\Gamma\left(k-\frac{r}{2}+\frac{1}{2}\right)}.$$

Thus

$$I_{k,\,m}(z) = \frac{z^m}{\Gamma(2m)} \sum_{r=0}^{\infty} \frac{z^{\frac{r}{2}}}{r!(2m+r)} \frac{\Gamma\left(k+\frac{r}{2}+\frac{1}{2}\right)}{\Gamma\left(k-\frac{r}{2}+\frac{1}{2}\right)} I_{k,\,\frac{r}{2}}(z). \tag{12}$$

This series converges absolutely and uniformly for all finite values of z, since the ratio test of convergence shows that u_{r+1}/u_r is of the order of $-z/2r$, where u_r is the rth term.

This series easily shows that $I_{k,m}(z)$ is a constant multiple of $I_{k,-m}(z)$ when $2m$ is an integer. It is therefore expected to lead to a simple limiting form for $N_{k,m}(z)$ when $2m$ is an integer.

Substituting from eq. (12) into eq. (7), the following expansion for this limiting form is obtained:

$$\begin{aligned}\pi N_{k,m}(z) = &\frac{\Gamma(\frac{1}{2}+m+k)}{\Gamma(\frac{1}{2}-m+k)} \\ &(\tfrac{1}{2}\psi(\tfrac{1}{2}+m+k)+\tfrac{1}{2}\psi(\tfrac{1}{2}-m+k)-\psi(2m)-\psi(2m+1)+\log z) I_{k,m}(z) \\ &-\frac{\Gamma(\frac{1}{2}+m+k)}{\Gamma(\frac{1}{2}-m+k)}\frac{z^m}{\Gamma(2m)}\sum_{r=0}^{\infty}\frac{z^{\frac{r}{2}}}{r!(2m+r)^2}\frac{\Gamma\left(\frac{1}{2}+\frac{r}{2}+k\right)}{\Gamma\left(\frac{1}{2}-\frac{r}{2}+k\right)} I_{k,\frac{r}{2}}(z) \\ &-\Gamma(2m+1)z^{-m}\sum_{r=0}^{\infty}\frac{z^{\frac{r}{2}}}{r!(2m-r)}\frac{\Gamma\left(\frac{1}{2}+\frac{r}{2}+k\right)}{\Gamma\left(\frac{1}{2}-\frac{r}{2}+k\right)} I_{k,\frac{r}{2}}(z),\end{aligned} \tag{13}$$

where in the last series the term $r = 2m$ is to be discarded.

5. The Coulomb Wave Functions for Atomic Electrons

If Coulomb's law applies for all values of r then the radial factor, multiplied by r, of the wave function of an atomic electron moving in the field of a nucleus of charge Ze is the regular solution $I_{k,m}(z)$ of eq. (3), provided that k, m and z satisfy eqs. (2). Owing to the finite size of the nucleus one has to modify Coulomb's law in the nuclear domain. In this case the wave function beyond the nucleus is a linear combination of the functions $I_{k,m}(z)$ and $N_{k,m}(z)$.

For an atomic electron $E < 0$, the values of k and z are purely real as inferred from (2). The functions $I_{k,m}(z)$ and $N_{k,m}(z)$ behave at infinity according to the law $\exp(z/2)$, so that when z is real they both tend to infinity as $z \to \infty$. It follows that, in general, there is no permissible combination of them which will remain finite at infinity. However, an exception arises when they are combined in the form of Whittaker's function $W_{k,m}(z)$ which is

known to remain finite at infinity. Whittaker and Watson introduced the following relation:

$$W_{k,m}(z)=\frac{\Gamma(-2m)}{\Gamma(\frac{1}{2}-m-k)}M_{k,m}(z)+\frac{\Gamma(2m)}{\Gamma(\frac{1}{2}+m-k)}M_{k,-m}(z)$$

provided that $2m$ is not an integer. This restriction can be removed by eliminating $M_{k,-m}(z)$ by means of eqs. (5) and (6), thus:

$$W_{k,m}(z)=\Gamma(\tfrac{1}{2}+m+k)\cos\pi\,(k-m-\tfrac{1}{2})I_{k,m}(z)+\Gamma(\tfrac{1}{2}-m+k)\sin\pi\,(k-m-\tfrac{1}{2})N_{k,m}(z), \quad (14)$$

which is valid for all m. This is the form in which $I_{k,m}(z)$ and $N_{k,m}(z)$ should be combined in order to give beyond the nucleus a well-behaved wave function for an atomic electron.

6. The Coulomb Wave Functions in Repulsive Fields

The radial equation of a particle moving under the influence of a Coulomb potential Ze^2/r is

$$\frac{\mathrm{d}^2W}{\mathrm{d}r^2}+\left\{\frac{2m}{\hbar^2}\left(E-\frac{Ze^2}{r}\right)-\frac{l(l+1)}{r^2}\right\}W=0. \quad (15)$$

By the following substitutions

$$E=\frac{me^4Z^2}{2\hbar^2k^2},\quad r=\frac{\hbar^2k}{2me^2Z}\zeta \text{ and } l=m-\tfrac{1}{2} \quad (16)$$

it transforms into

$$\frac{\mathrm{d}^2W}{\mathrm{d}\zeta^2}+\left\{\frac{1}{4}-\frac{k}{\zeta}+\frac{\frac{1}{4}-m^2}{\zeta^2}\right\}W=0. \quad (17)$$

When $2m$ is not an integer, two independent solutions of this equation are $I_{ik,m}(i\zeta)$ and $I_{ik,-m}(i\zeta)$.

Let $J_{k,m}(\zeta)$ denote the function

$$J_{k,m}(\zeta)=i^{-(\frac{1}{2}+m)}I_{ik,m}(i\zeta). \quad (18)$$

Substituting $k=ik$ and $z=i\zeta$, in eq. (11) the following expansion is obtained:

$$J_{k,m}(\zeta)=(\pi\zeta)^{\frac{1}{2}}\sum_{r=0}^{\infty}(-)^r\frac{i^{r+1}k\Gamma\left(ik+\frac{r}{2}\right)\zeta^{\frac{r}{2}}}{r!\Gamma\left(ik-\frac{r}{2}+1\right)\Gamma\left(\frac{1}{2}+m+\frac{r}{2}\right)}J_{m+\frac{r}{2}}(\tfrac{1}{2}\zeta)$$

$$=(\pi\zeta)^{\frac{1}{2}}\left\{\frac{1}{\Gamma(\frac{1}{2}+m)}J_m(\tfrac{1}{2}\zeta)+\frac{k\zeta^{\frac{1}{2}}}{1!\Gamma(1+m)}J_{m+\frac{1}{2}}(\tfrac{1}{2}\zeta)+\frac{k^2\zeta}{2!\Gamma(\frac{3}{2}+m)}J_{m+1}(\tfrac{1}{2}\zeta)+\ldots\right\}. \quad (19)$$

When k and ζ are real, this expansion involves no imaginaries. This clearance of imaginaries is the main advantage gained by this expansion over power series expansion.

When $2m$ is an integer

$$J_{k,-m}(\zeta) = e^{-i\pi m}\frac{\Gamma(\frac{1}{2}+m+ik)}{\Gamma(\frac{1}{2}-m+ik)}J_{k,m}(\zeta)$$

$$= \frac{1}{2\pi}\Gamma(\tfrac{1}{2}+m+ik)\Gamma(\tfrac{1}{2}+m-ik)(e^{\pi k}+(-)^{2m}e^{-\pi k})J_{k,m}(\zeta),$$

and therefore, in this case, a particular solution of eq. (17) has to be defined.

Let

$$Y_{k,m}(\zeta)$$

$$= \frac{\frac{1}{2\pi}\Gamma(\frac{1}{2}+m+ik)\Gamma(\frac{1}{2}+m-ik)(e^{\pi k}+e^{-\pi k}\cos 2\pi m)J_{k,m}(\zeta)-J_{k,-m}(\zeta)}{\sin 2\pi m} \qquad (20)$$

which is a solution of eq. (17) when $2m$ is not an integer. When $2m$ is an integer, $Y_{k,m}(\zeta)$ is defined by the limiting form of this equation.

In terms of $I_{ik,m}(i\zeta)$ and $N_{ik,m}(i\zeta)$, the function $Y_{k,m}(\zeta)$ is given by

$$Y_{k,m}(\zeta)=e^{i\frac{\pi}{2}(m-\frac{1}{2})}\left[\frac{1}{2\pi}\Gamma(\tfrac{1}{2}+m+ik)\Gamma(\tfrac{1}{2}+m-ik)e^{i\pi(m-\frac{1}{2}-ik)}I_{ik,m}(i\zeta)+N_{ik,m}(i\zeta)\right].$$

Using eqs. (13) and (18), $Y_{k,m}(\zeta)$ could be expanded, when $2m$ is an integer, as follows:

$$\pi Y_{k,m}(\zeta) = i^{2m}\frac{\Gamma(\frac{1}{2}+m+ik)}{\Gamma(\frac{1}{2}-m+ik)}\{\psi(\tfrac{1}{2}+m+ik)+\psi(\tfrac{1}{2}+m-ik)$$

$$+\log\zeta-\psi(2m)-\psi(2m+1)\}J_{k,m}(\zeta)$$

$$-i^{2m}\frac{\Gamma(\frac{1}{2}+m+ik)}{\Gamma(\frac{1}{2}-m+ik)}\frac{\zeta^m}{\Gamma(2m)}\sum_{r=0}^{\infty}\frac{\zeta^{\frac{r}{2}}}{r!(2m+r)^2}i^r\frac{\Gamma\left(\frac{1}{2}+\frac{r}{2}+ik\right)}{\Gamma\left(\frac{1}{2}-\frac{r}{2}+ik\right)}J_{k,\frac{r}{2}}(\zeta)$$

$$-\Gamma(2m+1)\zeta^{-m}\sum_{r=0}^{\infty}\frac{\zeta^{\frac{r}{2}}}{r!(2m-r)}i^r\frac{\Gamma\left(\frac{1}{2}+\frac{r}{2}+ik\right)}{\Gamma\left(\frac{1}{2}-\frac{r}{2}+ik\right)}J_{k,\frac{r}{2}}(\zeta), \qquad (21)$$

where in the last series the term $r = 2m$ is to be discarded. When k and ζ are real, this expansion does not involve any imaginaries. It is therefore convenient for calculation.

The functions $J_{k,m}(\zeta)$ and $Y_{k,m}(\zeta)$ form a complete system of solutions of eq. (17), when $2m$ is an integer.

In the usual discussions of nuclear collisions the repulsive inverse square field is modified in the nuclear domain. In these cases the wave functions beyond the nucleus are linear combinations of $J_{k,m}(\zeta)$ and $Y_{k,m}(\zeta)$. The expansions (19) and (21) are hoped to be useful in the computation of these functions.

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IMPRESSIONS OF THE SIXTH ROCHESTER CONFERENCE ON HIGH ENERGY NUCLEAR PHYSICS

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The annual Rochester Conferences, the first of which was organized by the University of Rochester in the winter of 1950/51, have become in these five years a tradition, the yearly event in high energy physics par excellence.

The number of participants was this year higher than ever, namely about 200, including about 50 from outside the United States.

This year's achievements seem, on the whole, to have been along lines already indicated during the foregoing conference. The main problems, and the theoretical approaches meant to deal with them, have not qualitatively changed, but progress has been made in the formal development of the theory. From the experimental side, one is struck more by the wealth of the results, the increasing accuracy, than by startling new discoveries.

Attention was centered around the interactions amongst the "classical" particles of nuclear physics (protons, neutrons and pions) and around interactions involving the "new" particles (heavy mesons and hyperons). Interactions of electrons, μ-mesons, and the electromagnetic field were mentioned only occasionally.

Nucleon-nucleon scattering

In relatively low-energy nucleon-nucleon scattering (below 500 MeV) the accent goes now more in the direction of a phase-shift analysis, rather than an attempt to fit scattering data — including polarization effects — by an interaction potential.

Experiments on high energy nucleon-nucleon scattering (up to 3 GeV) yielded information on the rates of single, double, triple, etc. meson production; in these processes an important role is played by the resonant state of the π-nucleon system ($T = \frac{3}{2}$, $P_{3/2}$),

which shows itself in, e.g., strong angular correlation between π^+ and p as products of a high-energy p—p-collision.

Pion-nucleon scattering

Scattering of π-mesons on protons has been studied up to energies of 4.5 GeV. Here also, at the higher energies, inelastic effects — production of additional mesons — are studied. The multiplicity seems to increase rather slowly with energy; Fermi's model and its modifications were discussed.

The phase shifts (as a function of energy) at the lower energies are not yet accurately determined. A fit, with help of an electronic computer, of π-nucleon-scattering data with only s- and p-wave phase shifts as functions of energy was not at all satisfactory.

At energies of the incident π-meson between 1 and 1.5 GeV d-wave scattering seems to be appreciable, as judged from angular distributions. One would conclude that the interaction region has a size of close to 10^{-13} cm.

The theoretical discussion of the phase-shifts is now done with the powerful apparatus of the so-called dispersion theory. With help of the "microscopic causality" condition it proved possible to relate the experimental phase-shifts to an effective coupling constant, which, in some way, represents the renormalized coupling between "dressed" nucleons and mesons. Such a coupling constant derived from the scattering data by use of the dispersion relations is now in good agreement with two values of the coupling constant obtained from analysis of the $T = \frac{3}{2}$ p-wave phase shifts with the effective range Chew-Low plot. This also agrees with the effective coupling constant derived from meson-photoproduction at threshold.

Incidentally, the optimistic opinion was occasionally expressed that this dispersion theory could be made the basis of a more general theory of elementary particles, giving up the too detailed description of interactions attempted by current field theories.

The π-nucleon cross section shows some resonance-like behaviour — besides the famous 200 MeV resonance corresponding to the $T = \frac{3}{2}$, $P_{3/2}$ state — at about 0.8 GeV and 1.5 GeV. The remark was made that these values correspond to nearly equidistant energies in the centre of mass system, and thus these "resonances" could conceivably be due to more than one particle interacting in the well-known $\frac{3}{2}$, $\frac{3}{2}$ resonant state.

Antiproton

The news from Berkeley concerning the anti-proton contained one rather unexpected item, namely the fact that absorption measurements in Be and Cu both lead to an absorption cross-section 50 % or more larger than the size of these nuclei.

Heavy mesons

The central problem in the domain of the new unstable particles is still that posed by the degeneracy in mass and life time of the different heavy mesons.

There seems to be little doubt, reviewing the 600 analysed τ-events, that the τ meson spin and parity are either 0^- or 1^+; and for either of these assignments decay into two π-mesons is forbidden. Still the masses for the $K_{\pi 2}$ and the τ, as well as their lifetimes, are equal within experimental errors.

The mass-degeneracy indicates that the two particles have similar strong interactions. This leads to a proposal of a new symmetry principle, operative in strong interactions. Particles which are produced, in Gell-Mann's scheme, in association with one (or an odd number) of heavy mesons should exist in two forms, with equal spins but opposite parities, just like the heavy mesons themselves. Experimentally, nothing is known about such a possible second Λ or Σ-particle. The life-time-degeneracy would require still another explanation, as this seems to refer to the weak interactions. A proposal of Lee and Orear (already published) has been disproved by the Berkeley group, at least insofar the τ meson cannot be the γ-parent of the $K_{\pi 2}$ (no γ's were found).

On the other hand, the Gell-Mann scheme has not been contradicted by the very extensive evidence. Particularly associated production, and also the difference in behaviour of K^+ and K^- particles in nuclear matter are strikingly confirmed by experiments at Brookhaven and at Berkeley. (A possible test of the connection of the Gell-Mann quantum number and isobaric spin has, by the way, been proposed: K^- capture in deuterium should yield $\Sigma^- + p$ and $\Sigma^0 + n$ in the ratio 2 : 1).

Hyperfragments

Discussion of hyperfragments leads to indications of strong spin-dependence of the forces between nucleons and Λ^0, and to a probable

assignment of spin and parity to the Λ particle ($\frac{1}{2}-$ or perhaps $\frac{1}{2}+$ or $\frac{3}{2}+$).

Weak interactions

The classification of the weak interactions is not satisfactory as yet. An attempt was made to extend isobaric spin conservation also to these, by introduction of a "particle" that would carry no energy or momentum, no angular momentum or charge, but only $\frac{1}{2}$ unit of isobaric spin (the "spurion").

Electromagnetic interactions

As far as non-nuclear interactions are concerned, the high energy electron scattering experiments on protons and deuterons were discussed, with the result that nucleons seem to have their charge and extra magnetic moments spread over a region of the order of 0.7×10^{-13} cm. Also an experiment was described, designed to observe pair creation of μ-mesons by γ-rays in the field of nuclei. The cross-section appears close to what one should expect with spin $\frac{1}{2}$ for the μ-meson.

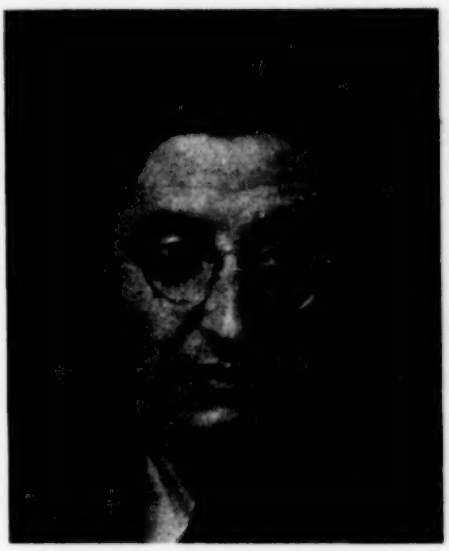
The above impressions do not claim any completeness whatsoever. The organizers of this conference have always taken pride in prompt, complete and careful publication of the conference proceedings. These should be consulted for a detailed and balanced account.

Acknowledgement

I am indepted to Dr R. Dalitz for a remark on the dispersion theory of the π-nucleon interaction.

Nuclear Physics, North-Holland Publishing Co., Amsterdam, **1 (1956)**

GEORGE PLACZEK (1905-1955)



George Placzek est mort le 9 octobre 1955 à Zürich après une pénible maladie. Ses nombreux amis pleurent en lui un homme d'une personnalité aussi profonde qu'attachante. Parmi ses collègues son nom restera connu comme celui d'un physicien théoricien de grande classe.

George Placzek, né à Brno (Tchécoslovaquie), fit ses études universitaires à Prague d'abord, puis à Vienne. Il s'oriente vers la physique théorique tout en manifestant un intérêt marqué pour le travail expérimental. Après un an passé à l'Université hébraïque de Jérusalem, il séjourne dans plusieurs villes universitaires européennes, Utrecht, Leipzig, Copenhague, Rome, Kharkow, Paris. En 1938 il se rend en Amérique où, après un séjour à Cornell University, il participe à l'effort de guerre dans le domaine de l'énergie nucléaire. Après les hostilités il est attaché pour deux ans environ au laboratoire de recherche de la General Electric Company à Schenectady. En 1948 il devient membre de l'Institute for Advanced Study de Princeton, où il travaillera jusqu'à quelques mois avant sa mort.

Les recherches de George Placzek se sont orientées durant sa vie entière vers des sujets se rattachant à la diffusion de la lumière et des particules par la matière. Dans ce domaine qui, sans se prêter à des découvertes spectaculaires, est cependant d'une importance capitale et d'une grande complexité, il avait acquis une étonnante maîtrise et c'est à juste titre qu'il était considéré par beaucoup comme la meilleure autorité sur le sujet. Il était malheureusement fort peu enclin à publier ses résultats et ne se sentait aucunement attiré par l'enseignement. C'est pour cette raison que seuls quelques collaborateurs immédiats ont, au cours des années, pu pleinement

tirer profit des connaissances et des qualités de physicien du disparu. L'auteur de ces lignes a eu le bonheur d'être l'un de ces privilégiés et quelques années de collaboration presque quotidienne avec George Placzek lui ont montré quel excellent maître pouvait être cet homme à l'abord si renfermé.

Les premiers travaux de Placzek ont porté sur la diffusion de la lumière par les molécules et sur l'effet Raman. Ils ont donné lieu entre autres à une publication d'ensemble devenue classique, l'article sur la diffusion Rayleigh et l'effet Raman dans le "Handbuch der Radiologie" (1934). Ses travaux sur la diffusion de la lumière l'avaient aussi amené à étudier la diffusion dans un liquide au voisinage du point critique (1930) et plus tard, en collaboration avec Landau, la structure fine (triplet) de diffusion d'une onde lumineuse monochromatique dans un liquide. Ce dernier travail, très incomplètement publié (1934), est analysé avec quelque détail par Frenkel dans son traité "Kinetic Theory of Liquids" (1946) †.

Dès leur début, les recherches consacrées à la diffusion nucléaire des neutrons lents attirèrent Placzek et il y participa à Rome et à Copenhague. Avec O. R. Frisch il étudia la capture des neutrons lents et, en 1937, il publia avec Bethe un article fondamental sur les réactions de résonance. La théorie générale des réactions nucléaires l'intéressa encore en 1939 èt des résultats obtenus à cette époque en collaboration avec N. Bohr et R. E. Peierls, quoique publiés sous forme très incomplète, sont maintenant considérés comme classiques.

Pendant les années de guerre George Placzek s'occupa du ralentissement des neutrons dans la matière, un problème de la plus haute importance pour la production de réactions neutroniques en chaîne. Quelques-unes de ses contributions à la théorie de ces phénomènes ont été publiées, et certains aspects mathématiques du problème font l'objet d'un ouvrage récent, rédigé en collaboration avec K. M. Case et F. de Hoffmann. Depuis la guerre enfin Placzek s'est intéressé de façon continue à l'étude détaillée de la diffusion des neutrons thermiques dans la matière, un problème dont l'intérêt immédiat est apparu lors de mesures destinées à déterminer expérimentalement l'interaction non électromagnétique entre neutrons et électrons, et dont l'intérêt plus lointain réside dans le fait qu'avec le progrès des techniques expérimentales les neutrons deviennent un

† Voyez aussi la thèse de R. S. Ketel (Groningen), Theorie van het Rayleigh-verstrooiingstriplet van vloeistoffen, M. Nijhoff, den Haag (1946) (L.R.).

puissant outil d'étude de la matière condensée (état solide, magnétisme, état liquide). Enfin dans les toutes dernières années Placzek s'était tourné aussi vers la mécanique statistique, spécialement la théorie des gaz quantiques, et avait obtenu des résultats intéressants sur la fonction de distribution radiale d'un gaz de sphères dures obéissant à la statistique de Bose-Einstein, pour les faibles densités. Il n'a malheureusement laissé à ce sujet que des notes extrêmement fragmentaires.

George Placzek avait, en plus d'une profonde connaissance de la physique, un jugement et un sens critique très sûrs. Autant dans son propre travail que dans celui des autres il aimait l'analyse serrée des faits et les arguments théoriques solidement fondés. Il avait un goût très vif pour l'élégance des déductions mathématiques et des calculs. Ses exigences sur tous ces points, et la complète maîtrise avec laquelle il y satisfaisait dans son propre travail, faisaient de lui un exemple tel qu'on en trouve peu parmi les physiciens actuels et sous ce rapport également il est regrettable qu'il n'ait pas fait école. Comme homme, Placzek cachait sous des dehors rudes et bardés de scepticisme une nature étonnamment captivante, sensible et délicate. Sa culture était très étendue, — dans certains domaines elle était de l'érudition —, et il en doublait l'attrait par le caractère très personnel de ses vues sur les sujets les plus variés. Au point de vue moral ses principes étaient sévères et cette qualité, jointe à l'expérience d'une vie exceptionnelle et à un esprit rigoureux de tolérance, faisait de lui un conseiller sûr et écouté. Il plaçait très haut le sens de l'amitié, et le plaisir si sincère qu'il éprouvait à revoir ses amis réchauffait le coeur. Pour tout cela il laissera chez tous ceux qui pleurent sa perte prématurée un souvenir ému et vivace.

Léon Van Hove

Principales Publications

Ueber die Lichtzerstreuung beim kritischen Punkt — Physik. Z. **31** (1930) 1052

Rayleigh Streuung und Raman Effekt — Handbuch der Radiologie, vol. **6** (1934)

(avec L. Landau) — Struktur der unverschobenen Streulinie — Phys. Z. d. Sowjetunion **5** (1934) 172

(avec H. A. Bethe) — Resonance effects in nuclear processes — Phys. Rev. **51** (1937) 450

(avec N. Bohr et R. Peierls) — Nuclear reactions in the continuous energy region — Nature **144** (1939) 200

On the theory of the slowing down of neutrons in heavy substances — Phys. Rev. **69** (1946) 423

(avec K. M. Case et F. de Hoffmann) — Introduction to the theory of neutron diffusion — Volume I (Los Alamos) 1953

(avec B. R. A. Nijboer et L. Van Hove) — Effect of short wavelength interference on neutron scattering by dense systems of heavy nuclei — Phys. Rev. **82** (1951) 392

The scattering of neutrons by systems of heavy nuclei — Phys. Rev. **86** (1951) 377

Incoherent neutron scattering by polycrystals — Phys. Rev. **93** (1954) 895

(avec L. Van Hove) — Crystal dynamics and inelastic scattering of neutrons — Phys. Rev. **93** (1954) 1207

NEWS ITEMS

CERN - FIRST ANNUAL REPORT

Amongst the measures approved at the Fifth Session of the CERN Council held in Geneva on the 1st June 1956, was the formal acceptance of the First Annual Report.

This Annual Report is unique in many ways. In particular it is the first report ever to be put out by an international organization devoted to a co-operative effort in the field of pure or fundamental scientific research.

A glance at the table of contents will inform the reader about the many aspects of the multitude of problems arising from the organization, planning and actual building of such an advanced project destined to carry on research and experimentation in the domain of high-energy nuclear physics. Dealt with in detail are the history and structure of CERN as a whole, the structure and membership of the Organization's Council, the internal organization of CERN itself, its Directorate-General and the various divisions. A list of the contributing Member States and their percentage shares in the budget appropriations is also published. Facts and figures regarding staff are given.

A brief chapter deals with the research activities carried out under the authority of the Director-General — theoretical work, work on nuclear magnetism, cosmic rays, etc.

Separate chapters deal with the six main divisions of CERN. These are: Proton Synchrotron; Synchro-cyclotron; Scientific and Technical Services; Site and Buildings; Theoretical Study Division; and finally, Administration.

Of particular interest are those chapters dealing with the two accelerating machines, the Proton Synchrotron and the Synchro-cyclotron. In these the reader may see for himself in both text and illustration the progress achieved in the construction of these advanced types of high-energy accelerators up to the end of December 1955.

It may be recalled that the Proton Synchrotron, designed for an energy of 25 GeV, is as big as any proton synchrotron at present under construction in the world (the machine at Brookhaven, the only other one comparable, is of the same order of acceleration energy).† The Synchro-cyclotron, though unique in its layout and general arrangement, is one of three of approximately the same energy range. There can be no doubt that, as is apparent from the Report, both the CERN machines have now reached a stage where construction will speed ever faster, despite the enormity of the problems involved in their planning and technical installation.

The following chapters deal with the activities of Scientific and Technical Services, Site and Buildings, and Theoretical Study.

Scientific and Technical Services is active largely in the design and building of ancillary electronic apparatus for experimental purposes, track chambers, planning and organization of the future CERN central workshop at Meyrin, and is responsible for the Information Service of the Organization. The Division has also helped to supervise the Cosmic Ray Research Group now working at the Jungfraujoch. In addition it has made a study of available types of electronic computers, installations for the liquefaction of gases and measures to be taken in the domain of protection against radiations.

The Site and Buildings Division, which supervises general layout and construction,

† The construction of a machine of 50—70 GeV acceleration energy is under study in the Soviet Union (L.R.).

including that of the two big accelerators, has had to carry out a correspondingly wide range of activities. Alone 1900 plans hitherto drawn up — exclusive of revised editions — have called for 150 000 man-hours of work. Other figures: 115 000 cubic metres of earth were excavated; 4.6 kilometres of pipes were laid; 11 000 cubic metres of reinforced concrete incorporating 9 900 tons of barytes — to increase the density of concrete for shielding purposes — and 569 tons of iron were set. The whole of this construction work required 428 000 man-hours.

The Theoretical Study Division, located in the Institute for Theoretical Physics of the University of Copenhagen, deals with scientific research on fundamental problems of nuclear physics. It also handles, in connection with the two CERN accelerators, theoretical problems relating to the focusing of ion beams in high-energy machines. The Division also trains young theoretical physicists and maintains active liaison with the laboratories of Liverpool and Uppsala.

Finally, the Administration Division, whose tasks are by nature manifold, co-ordinates the administrative work of all other divisions. It incorporates the closely-knit Finance Office, dealing with all aspects of financial policy and management, including the annual budget. The budget for 1955, it may be recalled, amounted to 25 112 000 Swiss Francs, subject to certain reservations. And in point of fact, since the specifications for certain buildings had not yet been approved and since there was some doubt regarding orders for certain materials during that financial year, expenditure was actually considerably less. The actual accounts for the financial year 1955, submitted at the June Council meeting, recorded a total expenditure of 18 839 279 Francs. The 1956 annual budget, providing for a total estimated expenditure of 39 600 000 Francs, will enjoy, however, the surplus remaining at the end of December 1955 so that it will only be necessary to call on CERN's twelve Member States for a total contribution of 33 000 000 Francs.

The Administration Division also incorporates a Personnel and Services Office, responsible for applying Staff Regulations and Rules, for staff recruiting and for personal matters generally. In this connection it should be pointed out that staff is required on an equitable basis, throughout the twelve Member States in Europe, corresponding in principle to the percentage contributions to the budget by the various countries.

The third department of the Administration Division, the Purchasing Office, is responsible for obtaining quotations and tenders, for contracts for materials and, of course, the purchase of supplies and instruments, etc., from firms in the twelve member countries in Europe.

The Annual Report includes a breakdown of 1955 expenditure, 1956 appropriations and the total capital investment programme for 1952—1960.

Four Appendices at the conclusion of the Report list CERN publications in general, CERN publications in the field of theoretical studies, lectures and colloquia of the Theoretical Study Division in Copenhagen and fourthly Colloquium lectures held jointly at the Institute of Physics of the University of Geneva under the auspices of CERN and the latter Institute.

CONFERENCE ON NUCLEAR STRUCTURE

The University of Pittsburgh announces its third "Conference on Nuclear Structure," to be held in Pittsburgh during June 6, 7, and 8 of 1957. Communications regarding the Conference should be addressed to N. Austern, at the Sarah Mellon Scaife Radiation Laboratory, University of Pittsburgh, Pittsburgh 13, Pennsylvania, U.S.A. It is expected that this Conference will be held regularly thereafter at two-year intervals.

FOR PROSPECTIVE CONTRIBUTORS

Since the nomenclature in the field of Nuclear Physics is not yet quite unified and a number of illogical expressions are unfortunately still widely used, it is advisable to give the following indications about the terminology adopted in this journal.

(1) The word 'electron' applies to particles with an elementary charge of either sign. When it is desirable to emphasize the sign of the charge, the words 'positon' and 'negaton' are used. Thus the illogical phrase 'electron-positron pair' is replaced by 'electron pair', or, if required, by 'positon-negaton pair'.

(2) Nuclei (or nucleids, *not* 'nuclids') are conveniently divided according to the parity of the proton and neutron numbers Z, N, into four families as follows:

even mass nuclei	even nuclei (Z even, N even) odd nuclei (Z odd, N odd)
odd mass nuclei	odd proton nuclei (Z odd, N even) odd neutron nuclei (Z even, N odd).

(3) The expressions 'isotopic spin', 'isotopic quantum number' are replaced by 'isobaric spin', 'isobaric quantum number'.

(4) For the elementary particles use is made of the latest nomenclature adopted by international agreement.

Editors and publishers have taken steps to secure prompt publication of accepted contributions; to achieve this end they count on the co-operation of authors, who are expected, as a matter of course, to submit neatly prepared typescripts of well-written papers and to return carefully corrected proofs without delay.

QUANTUM FIELD THEORY

BY HIROOMI UMEZAWA

Professor in Theoretical Physics, Tokyo

The aim of this book is to present comprehensive treatises about relativistic field equations, their quantisation, renormalisation theory, scattering problems, methods of calculations and various specialized problems of known particles and their interactions.

1956 – 6 × 9 – XVI + 350 pages – f 35,– $ 9.25 £ 3.10.0

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NORTH-HOLLAND PUBLISHING COMPANY

STARTING
PUBLICATION IN FALL 1956

Pre-Announcement

NUCLEAR INSTRUMENTS

A journal on accelerators, instrumentation and techniques in nuclear physics

Editor: KAI SIEGBAHN, University of Uppsala, Sweden

NUCLEAR INSTRUMENTS will publish experimental, theoretical and technical articles on subjects like: accelerators, isotope separators and mass spectrometers, instruments for nuclear spectroscopy, counters and detectors of nuclear radiation, electronic equipment used for nuclear instrumentation and automation, miscellaneous nuclear techniques like the preparation of targets or high specific activity sources, magnetic field measurements and vacuum technique with particular bearing upon nuclear apparatus etc. Furthermore, articles will be accepted dealing with dosimetry, and with the instruments used in tracer technique. Survey articles, letters to the editor, short laboratory notes and book reviews will also be included.

NUCLEAR INSTRUMENTS is strictly international in character. The Editorial Board consists of nuclear physicists representing all countries active in this field.

Details of publication will be announced later.

NORTH-HOLLAND PUBLISHING COMPANY